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2003 Q16

16. Ali, Bonnie, Carlo and Dianna are going to drive together to a nearby theme park. The car they are using has four seats: one driver's seat, one front passenger seat and two back seats. Bonnie and Carlo are the only two who can drive the car. How many possible seating arrangements are there?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 12 **(E)** 24

16. **(D)** There are 2 choices for the driver. The other three can seat themselves in $3 \times 2 \times 1 = 6$ different ways. So the number of seating arrangements is $2 \times 6 = 12$.

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2012 Q16

16. Each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 is used only once to make two five-digit numbers so that they have the largest possible sum. Which of the following could be one of the numbers?

(A) 76531 **(B)** 86724 **(C)** 87431 **(D)** 96240 **(E)** 97403

16. **Answer (C):** The sum will be as large as possible when the largest digits are placed in the most significant places. The 8 and 9 should be in the ten-thousands place, the 6 and 7 in the thousands place, the 4 and 5 in the hundreds place, the 2 and 3 in the tens place, and the 0 and 1 in the units place. The only choice that fits that description is 87431. (In this case the other number is 96520, giving the largest sum of 183951.)

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2014 Q16

16. The "Middle School Eight" basketball conference has 8 teams. Every season, each team plays every other conference team twice (home and away), and each team also plays 4 games against non-conference opponents. What is the total number of games in a season involving "Middle School Eight" teams?

(A) 60 **(B)** 88 **(C)** 96 **(D)** 144 **(E)** 160

16. **Answer (B):** Each team plays 4 non-conference games for a total of 32 games against non-conference opponents. Each team plays 7 conference games at home for a total of 56 games within the conference. The total number of games is $32 + 56 = 88$.

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2008 Q17

17. Ms. Osborne asks each student in her class to draw a rectangle with integer side lengths and a perimeter of 50 units. All of her students calculate the area of the rectangle they draw. What is the difference between the largest and smallest possible areas of the rectangles?

(A) 76 **(B)** 120 **(C)** 128 **(D)** 132 **(E)** 136

17. **Answer (D):** The formula for the perimeter of a rectangle is $2l + 2w$, so $2l + 2w = 50$, and $l + w = 25$. Make a chart of the possible widths, lengths, and areas, assuming all the widths are shorter than all the lengths.

Width	1	2	3	4	5	6	7	8	9	10	11	12
Length	24	23	22	21	20	19	18	17	16	15	14	13
Area	24	46	66	84	100	114	126	136	144	150	154	156

The largest possible area is $13 \times 12 = 156$ and the smallest is $1 \times 24 = 24$, for a difference of $156 - 24 = 132$ square units.
Note: The product of two numbers with a fixed sum increases as the numbers get closer together. That means, given the same perimeter, the square has a larger area than any rectangle, and a rectangle with a shape closest to a square will have a larger area than other rectangles with equal perimeters.

2004 Q17

17. Three friends have a total of 6 identical pencils, and each one has at least one pencil. In how many ways can this happen?

(A) 1 **(B)** 3 **(C)** 6 **(D)** 10 **(E)** 12

17. **(D)** The largest number of pencils that any friend can have is four. There are 3 ways that this can happen: $(4, 1, 1)$, $(1, 4, 1)$ and $(1, 1, 4)$. There are 6 ways one person can have 3 pencils: $(3, 2, 1)$, $(3, 1, 2)$, $(2, 3, 1)$, $(2, 1, 3)$, $(1, 2, 3)$ and $(1, 3, 2)$. There is only one way all three can have two pencils each: $(2, 2, 2)$. The total number of possibilities is $3 + 6 + 1 = 10$.

OR

The possible distributions of 6 pencils among 3 friends are the following:

1	1	4
1	2	3
1	3	2
1	4	1
2	1	3
2	2	2
2	3	1
3	1	2
3	2	1
4	1	1

The number of possible distributions is 10.

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2016 Q18

18. In an All-Area track meet, 216 sprinters enter a 100-meter dash competition. The track has 6 lanes, so only 6 sprinters can compete at a time. At the end of each race the five non-winners are eliminated, and the winner will compete again in a later race. How many races are needed to determine the champion sprinter?

(A) 36 **(B)** 42 **(C)** 43 **(D)** 60 **(E)** 72

18. Answer (C):

Divide the 216 sprinters into 36 groups of 6. Run 36 races to eliminate 180 sprinters, leaving 36 winners. Divide the 36 winners into 6 groups of 6, run 6 races to eliminate 30 sprinters, leaving 6 winners. Finally run the last race to determine the champion. The number of races run is $36+6+1=43$.

OR

When all the races have been run, 215 sprinters will have been eliminated. Since 5 sprinters are eliminated in each race, there are $\frac{215}{5}=43$ races needed to determine the champion.

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2013 Q19

19. Bridget, Cassie, and Hannah are discussing the results of their last math test. Hannah shows Bridget and Cassie her test, but Bridget and Cassie don't show their tests to anyone. Cassie says, "I didn't get the lowest score in our class," and Bridget adds, "I didn't get the highest score." What is the ranking of the three girls from highest to lowest?

(A) Hannah, Cassie, Bridget **(B)** Hannah, Bridget, Cassie

(C) Cassie, Bridget, Hannah **(D)** Cassie, Hannah, Bridget

(E) Bridget, Cassie, Hannah

19. **Answer (D):** Cassie says, "I didn't get the lowest score," so her score is higher than Hannah's score. Bridget says, "I didn't get the highest score," so her score is lower than Hannah's score. Therefore the order, from highest to lowest, must be Cassie, Hannah, Bridget.

1990 Q19

19 . There are 120 seats in a row. What is the fewest number of seats that must be occupied so the next person to be seated must sit next to someone?

A) 30 B) 40 C) 41 D) 60 E) 119

19. B In order for the fewest number of seats to be occupied, there must be someone in every third seat, beginning with #2 and ending with #119. There are a total of $\frac{120}{3} = 40$ occupied seats.

OR

Consider some simpler cases and make a table:

Number of seats in the row:	3	6	9	12
Number of occupied seats in the row:	1	2	3	4

In each case, the middle seat in every group of three seats must be occupied, so the desired number of occupied seats in a row of 120 seats is $\frac{120}{3} = 40$.

1991 Q20

20. In the addition problem, each digit has been replaced by a letter. If different letters represent different digits then $C =$

$$\begin{array}{r} A\ B\ C \\ A\ B \\ +\quad\ \ A \\ \hline 3\ 0\ 0 \end{array}$$

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

20. (A) For the sum to be 300, $A = 2$ in the hundreds' place, since $A = 1$ gives numbers too small and $A = 3, 4, \ldots$ makes the sum too large. If $A = 2$ then $B = 7$, since $A = 2$ and $B = 8$ or 9 would be too large for the ones' place and $A = 2$ and $B = 6, 5, \ldots$ would not be enough to carry a 1 from the tens' to the hundreds' place. Thus, if $A = 2$ and $B = 7$ and $A + B + C = 10$ in the ones' place, then $C = 1$.

OR

Since the second column carry is 1, $A = 2$. The first column carry is also 1 since the sum of B and C is less than or equal to 17 and $A = 2$. In the second column, this makes $A + B +$ (first column carry) $= 2 + B + 1 = 10$, so $B = 7$. Then from the first column $A + B + C = 2 + 7 + C = 10$, so $C = 1$.

1994 Q20

20. Let W, X, Y and Z be four different digits selected from the set

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

If the sum $\frac{W}{X} + \frac{Y}{Z}$ is to be as small as possible, then $\frac{W}{X} + \frac{Y}{Z}$ must equal

(A) $\frac{2}{17}$ (B) $\frac{3}{17}$ (C) $\frac{17}{72}$ (D) $\frac{25}{72}$ (E) $\frac{13}{36}$

20. **(D)** Small numerators and large denominators yield small fractions. Use 1 and 2 for numerators and 8 and 9 for denominators to obtain the smallest fractions, then compare the sums

$$\frac{1}{8}+\frac{2}{9}=\frac{9+16}{72}=\frac{25}{72} \quad \text{and} \quad \frac{1}{9}+\frac{2}{8}=\frac{8+18}{72}=\frac{26}{72}$$

to see that $\frac{25}{72}$ is the answer.

Note. Analyzing the sums yields $\frac{1}{8}+\frac{1}{9}+\frac{1}{9}$ which is smaller than $\frac{1}{9}+\frac{1}{8}+\frac{1}{8}$.

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2006 Q20

20. A singles tournament had six players. Each player played every other player only once, with no ties. If Helen won 4 games, Ines won 3 games, Janet won 2 games, Kendra won 2 games and Lara won 2 games, how many games did Monica win?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

20. **(C)** Each of the six players played 5 games, and each game involved two players. So there were $\frac{6 \cdot 5}{2}=15$ games. Helen, Ines, Janet, Kendra and Lara won a total of $4+3+2+2+2=13$ games, so Monica won $15-13=2$ games.