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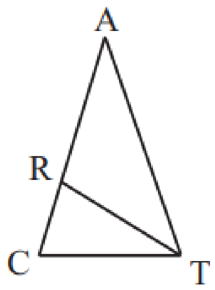
2000 Q13

13. In triangle CAT, we have $\angle ACT = \angle ATC$ and $\angle CAT = 36^\circ$. If $\overline{TR}$ bisects $\angle ATC$, then $\angle CRT =$

(A) 16° **(B)** 51° **(C)** 72° **(D)** 90° **(E)** 108°



13. **Answer (C):** Since $\angle ACT = \angle ATC$ and $\angle CAT = 36^\circ$, we have $2(\angle ATC) = 180^\circ - 36^\circ = 144^\circ$ and $\angle ATC = \angle ACT = 72^\circ$. Because $\overline{TR}$ bisects $\angle ATC$, $\angle CTR = \frac{1}{2}(72^\circ) = 36^\circ$. In triangle CRT, $\angle CRT = 180^\circ - 36^\circ - 72^\circ = 72^\circ$. Note that some texts use $\angle ACT$ to define the angle and $m\angle ACT$ to indicate its measure.

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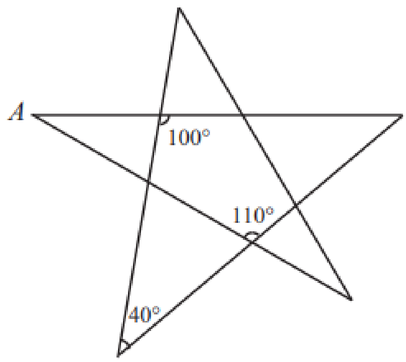
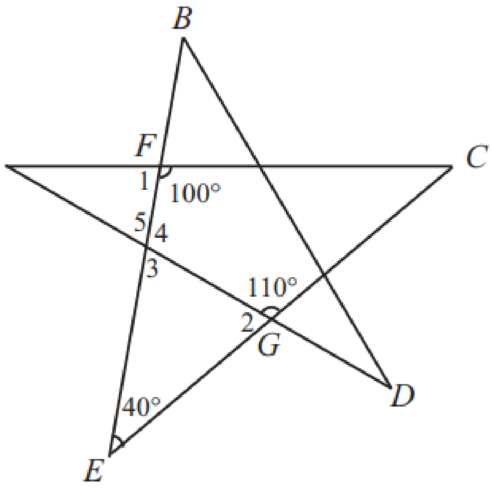
1999 Q21

21. The degree measure of angle A is

(A) 20 **(B)** 30 **(C)** 35 **(D)** 40 **(E)** 45



21. **Answer (B):** Since $\angle 1$ forms a straight line with angle $100°$, $\angle 1 = 80°$. Since $\angle 2$ forms a straight line with angle $110°$, $\angle 2 = 70°$. Angle 3 is the third angle in a triangle with $\angle E = 40°$ and $\angle 2 = 70°$, so $\angle 3 = 180° - 40° - 70° = 70°$.Angle $4 = 110°$ since it forms a straight angle with $\angle 3$. Then $\angle 5$ forms a straight angle with $\angle 4$, so $\angle 5 = 70°$. (Or $\angle 3 = \angle 5$ because they are vertical angles.) Therefore, $\angle A = 180° - \angle 1 - \angle 5 = 180° - 80° - 70° = 30°$.

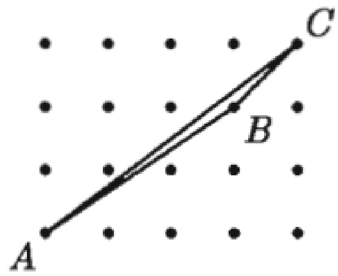


OR

The angle sum in $\triangle CEF$ is $180°$, so $\angle C = 180° - 40° - 100° = 40°$. In $\triangle ACG$, $\angle G = 110°$ and $\angle C = 40°$, so $\angle A = 180° - 110° - 40° = 30°$.

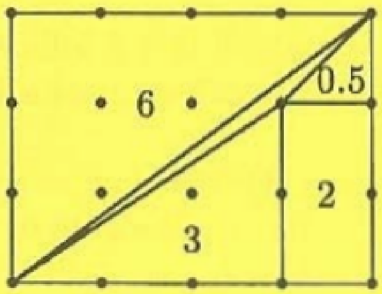
1996 Q22

22. The horizontal and vertical distances between adjacent points equal 1 unit. The area of triangle ABC is



(A) 1/4 **(B)** 1/2 **(C)** 3/4 **(D)** 1 **(E)** 5/4

22. **(B)** From the total area of 12, subtract the areas of the four surrounding polygons whose areas are indicated in the diagram. Thus the area of the remaining triangle ABC is

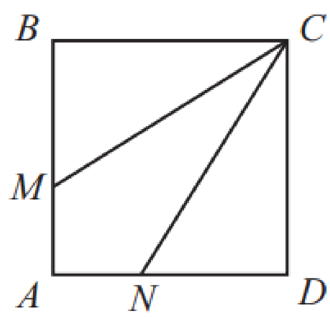


$$12 - 6 - 3 - 0.5 - 2 = 0.5 = 1/2.$$

Note. Points whose coordinates are integers are called *lattice* points. According to Pick's Theorem, if there are I lattice points in the interior of a triangle and B lattice points on the boundary, then the area of the triangle is $I + B/2 - 1$. In this problem, $I = 0$ and $B = 3$. Therefore the area of the triangle is $0 + 3/2 - 1 = 1/2$.

1999 Q23

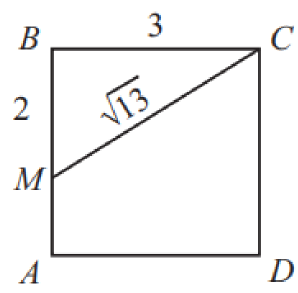
23. Square $ABCD$ has sides of length 3. Segments CM and CN divide the square's area into three equal part. How long is segment CM ?



(A) $\sqrt{10}$ **(B)** $\sqrt{12}$ **(C)** $\sqrt{13}$

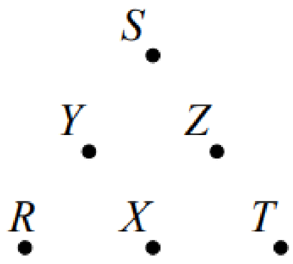
(D) $\sqrt{14}$ **(E)** $\sqrt{15}$

23. **Answer (C):** One-third of the square's area is 3, so triangle MBC has area $3 = \frac{1}{2}(MB)(BC)$. Since side BC is 3, side MB must be 2. The hypotenuse CM of this right triangle is $\sqrt{2^2 + 3^2} = \sqrt{13}$.



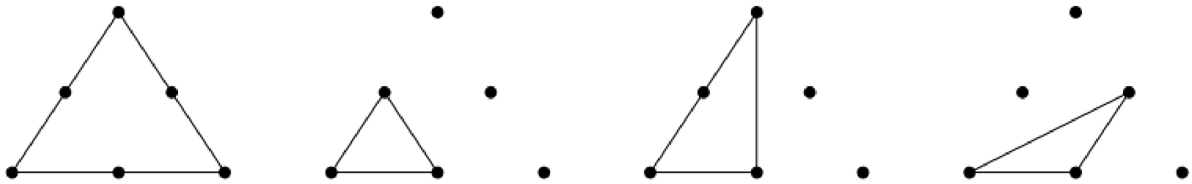
2001 Q23

23. Points R, S and T are vertices of an equilateral triangle, and points X, Y and Z are midpoints of its sides. How many noncongruent triangles can be drawn using any three of these six points as vertices?



(A) 1 (B) 2 (C) 3 (D) 4 (E) 20

23. (D) There are four noncongruent triangles.

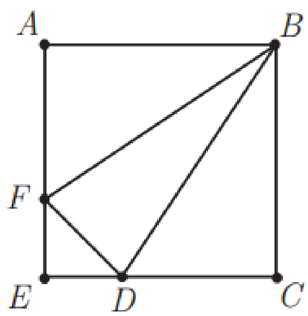


OR

The seventeen possible triangles may be divided into four congruence classes: {RST}; {RXY, XTZ, YZS, XYZ}; {RXS, TXS, RZS, RZT, TYR, TYS}; {RXZ, RYZ, TXY, TZY, XYS, XZS}

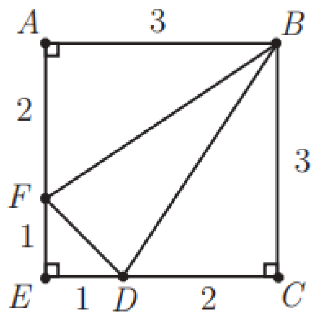
2008 Q23

23. In square $ABCE$, $AF = 2FE$ and $CD = 2DE$. What is the ratio of the area of $\triangle BFD$ to the area of square $ABCE$?



(A) $\frac{1}{6}$ **(B)** $\frac{2}{9}$ **(C)** $\frac{5}{18}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{7}{20}$

23. **Answer (C):** Because the answer is a ratio, it does not depend on the side length of the square. Let $AF = 2$ and $FE = 1$. That means square $ABCE$ has side length 3 and area $3^2 = 9$ square units. The area of $\triangle BAF$ is equal to the area of $\triangle BCD = \frac{1}{2} \cdot 3 \cdot 2 = 3$ square units. Triangle DEF is an isosceles right triangle with leg lengths $DE = FE = 1$. The area of $\triangle DEF$ is $\frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$ square units. The area of $\triangle BFD$ is equal to the area of the square minus the areas of the three right triangles: $9 - (3 + 3 + \frac{1}{2}) = \frac{5}{2}$. So the ratio of the area of $\triangle BFD$ to the area of square $ABCE$ is $\frac{\frac{5}{2}}{9} = \frac{5}{18}$.



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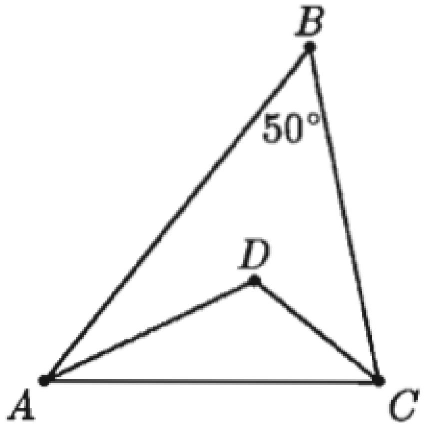
1996 Q24

24. The measure of angle ABC is $50°$, $\overline{AD}$ bisects angle BAC, and $\overline{DC}$ bisects angle BCA. The measure of angle ADC is

(A) 90° **(B)** 100° **(C)** 115°
(D) 122.5° **(E)** 125°



24. (C) Since the sum of the measures of the angles of a triangle is 180°, in triangle ABC it follows that

$$\angle BAC + \angle BCA = 180° - 50° = 130°.$$

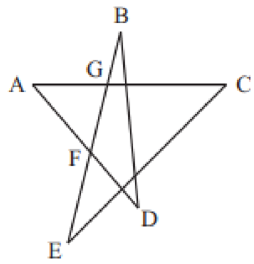
The measures of angles DAC and DCA are half that of angles BAC and BCA, respectively, so

$$\angle DAC + \angle DCA = \frac{130°}{2} = 65°.$$

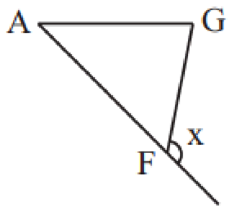
In triangle ACD, we have $\angle ADC = 180° - 65° = 115°$.

2000 Q24

24. If $\angle A = 20^\circ$ and $\angle AFG = \angle AGF$, Then $\angle B + \angle D =$



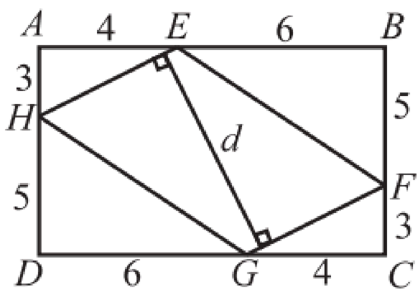
(A) 48° **(B)** 60° **(C)** 72° **(D)** 80° **(E)** 90°



24. **Answer (D):** Since $\angle AFG = \angle AGF$ and $\angle GAF + \angle AFG + \angle AGF = 180^\circ$, we have $20^\circ + 2(\angle AFG) = 180^\circ$. So $\angle AFG = 80^\circ$. Also, $\angle AFG + \angle BFD = 190^\circ$, so $\angle BFD = 100^\circ$. The sum of the angles of $\triangle BFD$ is 180°, so $\angle B + \angle D = 80^\circ$.
Note: In $\triangle AFG, \angle AFG = \angle B + \angle D$. In general, an exterior angle of a triangle equals the sum of its remote interior angles. For example, in $\triangle GAF, \angle x = \angle GAF + \angle AGF$.
Note that, as in Problem 13, some texts use different symbols to represent an angle and its degree measure.

2004 Q24

24. In the figure, $ABCD$ is a rectangle and $EFGH$ is a parallelogram. Using the measurements given in the figure, what is the length d of the segment that is perpendicular to $\overline{HE}$ and $\overline{FG}$?



(A) 6.8 **(B)** 7.1 **(C)** 7.6 **(D)** 7.8 **(E)** 8.1

24. **(C)** By the Pythagorean Theorem, $HE = 5$. Rectangle $ABCD$ has area $10 \times 8 = 80$, and the corner triangles have areas $\frac{1}{2} \times 3 \times 4 = 6$ and $\frac{1}{2} \times 6 \times 5 = 15$. So the area of $EFGH$ is $80 - (2)(6) - (2)(15) = 38$. Because the area of $EFGH$ is $EH \times d$ and $EH = 5$, $38 = 5 \times d$, so $d = 7.6$.

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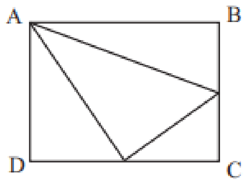
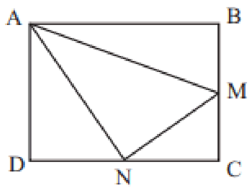
2000 Q25

25. The area of rectangle $ABCD$ is 72. If point A and the midpoints of $\overline{BC}$ and $\overline{CD}$ are joined to form a triangle, the area of that triangle is



(A) 21 **(B)** 27 **(C)** 30 **(D)** 36 **(E)** 40

25. **Answer (B):** Three right triangles lie outside $\triangle AMN$. Their areas are $\frac{1}{4}$, $\frac{1}{4}$, and $\frac{1}{8}$ for a total of $\frac{5}{8}$ of the rectangle. The area of $\triangle AMN$ is $\frac{3}{8}(72) = 27$.



OR

Let the rectangle have sides of $2a$ and $2b$ so that $4ab = 72$ and $ab = 18$.Three right triangles lie outside triangle AMN, and their areas are $\frac{1}{2}(2a)(b)$, $\frac{1}{2}(2b)(a)$, $\frac{1}{2}(a)(b)$, for a total of $\frac{5}{2}(ab) = \frac{5}{2}(18) = 45$. The area of triangle AMN is $72 - 45 = 27$.