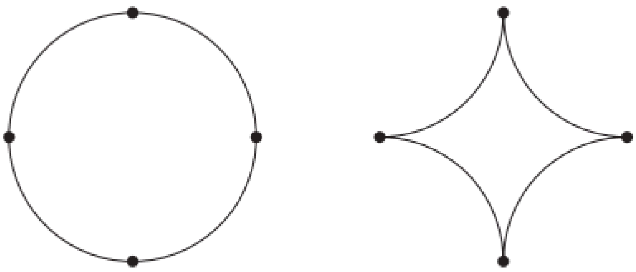
2012 Q24

24. A circle of radius 2 is cut into four congruent arcs. The four arcs are joined to form the star figure shown. What is the ratio of the area of the star figure to the area of the original circle?

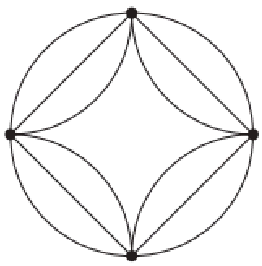


(A) $\frac{4-\pi}{\pi}$ **(B)** $\frac{1}{\pi}$ **(C)** $\frac{\sqrt{2}}{\pi}$ **(D)** $\frac{\pi-1}{\pi}$ **(E)** $\frac{3}{\pi}$

24. **Answer (A):** Translate the star into the circle so that the points of the star coincide with the points on the circle. Construct four segments connecting the consecutive points of the circle and the star, creating a square concentric to the circle.

The area of the circle is $\pi(2)^2 = 4\pi$. The square is made up of four congruent right triangles with area $\frac{1}{2}(2 \times 2) = 2$, so the area of the square is $4 \times 2 = 8$. The area inside the circle but outside the square is $4\pi - 8$.

This is also the area inside the square but outside the star. So, the area of the star is $8 - (4\pi - 8) = 16 - 4\pi$. The ratio of the area of the star figure to the area of the original circle is $\frac{16-4\pi}{4\pi} = \frac{4-\pi}{\pi}$.

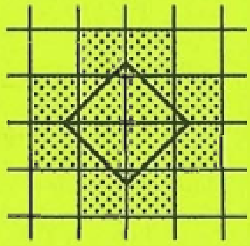


1993 Q25

25. A checkerboard consists of one-inch squares. A square card, 1.5 inches on a side, is placed on the board so that it covers part or all of the area of each of n squares. The maximum possible value of n is

(A) 4 or 5 (B) 6 or 7 (C) 8 or 9 (D) 10 or 11 (E) 12 or more

25. **(E)** Using the Pythagorean Theorem, the length of the diagonal of the card is $\sqrt{(1.5)^2+(1.5)^2} = \sqrt{4.5} \approx 2.1$. This is longer than 2, the length of two adjacent squares. The figure shows 12 squares being touched.



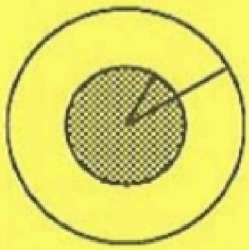
Query. Is 12 the maximum number of squares that can be touched?

1996 Q25

25. A point is chosen at random from within a circular region. What is the probability that the point is closer to the center of the region than it is to the boundary of the region?

(A) 1/4 (B) 1/3 (C) 1/2 (D) 2/3 (E) 3/4

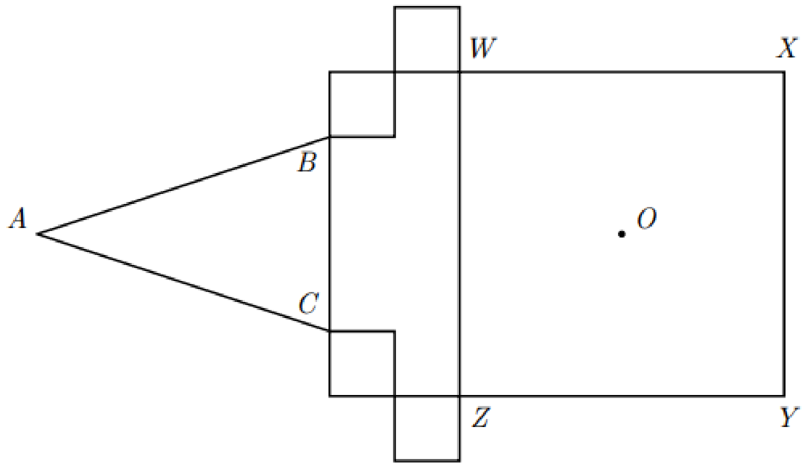
25. **(A)** Suppose that the circle has radius 1. Then, being closer to the center of the region than the boundary of the region (the circle) would mean the chosen point must be inside the circle of radius 1/2 with the same center as the larger circle of radius 1. The area of the smaller region is $\pi(1/2)^2 = \pi/4$, and the area of the total region is $\pi(1)^2 = \pi$. Since the area of the smaller region is 1/4 of the area of the total region, the required probability is 1/4.



Note. The odds that the point is closer to the center are 1 : 3.

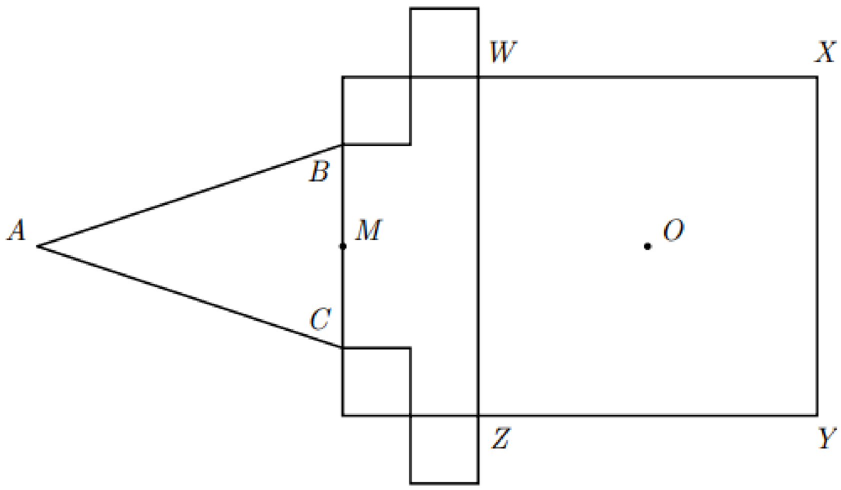
2003 Q25

25. In the figure, the area of square $WXYZ$ is 25 cm^2. The four smaller squares have sides 1 cm long, either parallel to or coinciding with the sides of the large square. In $\triangle ABC$, $AB = AC$, and when $\triangle ABC$ is folded over side $\overline{BC}$, point A coincides with O, the center of square $WXYZ$. What is the area of $\triangle ABC$, in square centimeters?



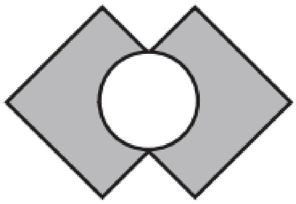
(A) $\frac{15}{4}$ **(B)** $\frac{21}{4}$ **(C)** $\frac{27}{4}$ **(D)** $\frac{21}{2}$ **(E)** $\frac{27}{2}$

25. **(C)** Let M be the midpoint of $\overline{BC}$. Since $\triangle ABC$ is isosceles, $\overline{AM}$ is an altitude to base $\overline{BC}$. Because A coincides with O when $\triangle ABC$ is folded along $\overline{BC}$, it follows that $AM = MO = \frac{5}{2} + 1 + 1 = \frac{9}{2}$ cm. Also, $BC = 5 - 1 - 1 = 3$ cm, so the area of $\triangle ABC$ is $\frac{1}{2} \cdot BC \cdot AM = \frac{1}{2} \cdot 3 \cdot \frac{9}{2} = \frac{27}{4}$ cm^2.



2004 Q25

25. Two 4×4 squares intersect at right angles, bisecting their intersecting sides, as shown. The circle's diameter is the segment between the two points of intersection. What is the area of the shaded region created by removing the circle from the squares?



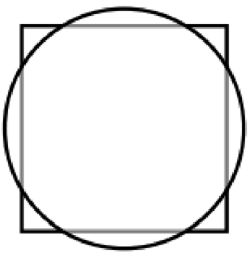
(A) $16 - 4\pi$ **(B)** $16 - 2\pi$ **(C)** $28 - 4\pi$ **(D)** $28 - 2\pi$ **(E)** $32 - 2\pi$

25. **(D)** The overlap of the two squares is a smaller square with side length 2, so the area of the region covered by the squares is $2(4 \times 4) - (2 \times 2) = 32 - 4 = 28$. The diameter of the circle has length $\sqrt{2^2 + 2^2} = \sqrt{8}$, the length of the diagonal of the smaller square. The shaded area created by removing the circle from the squares is $28 - \pi\left(\frac{\sqrt{8}}{2}\right)^2 = 28 - 2\pi$.

6 / 11

2005 Q25

25. A square with side length 2 and a circle share the same center. The total area of the regions that are inside the circle and outside the square is equal to the total area of the regions that are outside the circle and inside the square. What is the radius of the circle?



(A) $\frac{2}{\sqrt{\pi}}$ **(B)** $\frac{1+\sqrt{2}}{2}$ **(C)** $\frac{3}{2}$ **(D)** $\sqrt{3}$ **(E)** $\sqrt{\pi}$

25. **(A)** Because the circle and square share the same interior region and the area of the two exterior regions indicated are equal, the square and the circle must have equal area. The area of the square is 2^2 or 4. Because the area of both the circle and the square is 4, $4 = \pi r^2$. Solving for r, the radius of the circle, yields $r^2 = \frac{4}{\pi}$, so $r = \sqrt{\frac{4}{\pi}} = \frac{2}{\sqrt{\pi}}$.

Note: It is not necessary that the circle and square have the same center.

7 / 11

2008 Q25

25. Margie's winning art design is shown. The smallest circle has radius 2 inches, with each successive circle's radius increasing by 2 inches. Approximately what percent of the design is black?

(A) 42 **(B)** 44 **(C)** 45 **(D)** 46 **(E)** 48

25. **Answer (A):**

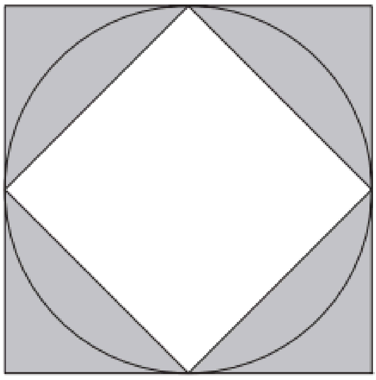
Circle #	Radius	Area
1	2	4π
2	4	16π
3	6	36π
4	8	64π
5	10	100π
6	12	144π

The total black area is $4\pi + (36 - 16)\pi + (100 - 64)\pi = 60\pi$ in^2.
So the percent of the design that is black is $100 \times \frac{60\pi}{144\pi} = 100 \times \frac{5}{12}$ or about 42%.

8 / 11

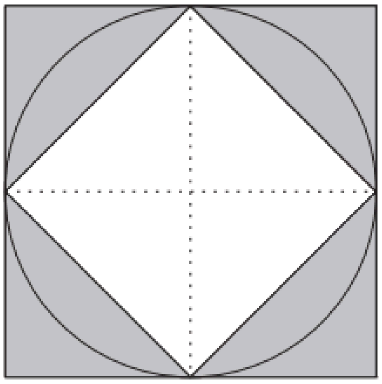
2011 Q25

25. A circle with radius 1 is inscribed in a square and circumscribed about another square as shown. Which fraction is closest to the ratio of the circle's shaded area to the shaded area between the two squares?



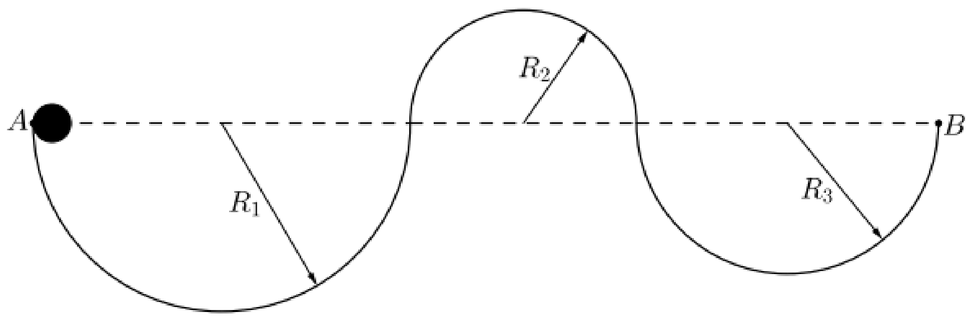
(A) $\frac{1}{2}$ **(B)** 1 **(C)** $\frac{3}{2}$ **(D)** 2 **(E)** $\frac{5}{2}$

25. **Answer (A):** The area of a circle of radius 1 is $\pi(1)^2 = \pi$. The side length of the big square is the diameter of the circle, which is 2, so its area is $2^2 = 4$. The big square can be divided into 8 congruent triangles, and the shaded area is made up of 4 of those triangles. The shaded area is half the area of the big square, which is 2. The requested ratio of the two shaded areas is $\frac{\pi-2}{2} \approx \frac{3.14-2}{2} \approx \frac{1}{2}$.



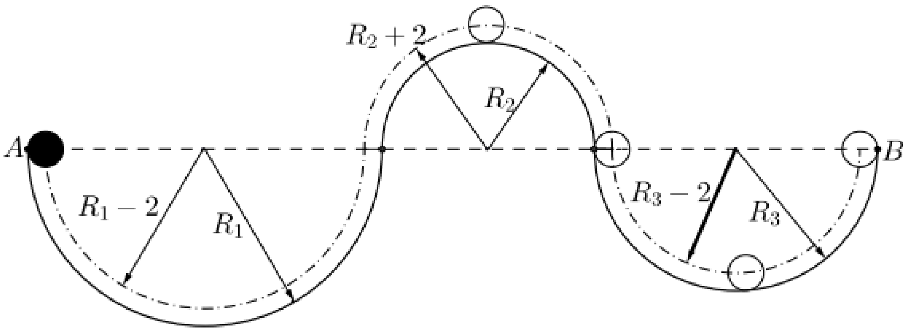
2013 Q25

25. A ball with diameter 4 inches starts at point A to roll along the track shown. The track is comprised of 3 semicircular arcs whose radii are $R_1 = 100$ inches, $R_2 = 60$ inches, and $R_3 = 80$ inches, respectively. The ball always remains in contact with the track and does not slip. What is the distance in inches the center of the ball travels over the course from A to B?



(A) 238π **(B)** 240π **(C)** 260π **(D)** 280π **(E)** 500π

25. **Answer (A):** The diameter of the ball is 4 inches, so its radius is 2 inches. The center of the ball rolls through semicircles of radii $R_1 - 2 = 100 - 2 = 98$ inches, $R_2 + 2 = 60 + 2 = 62$ inches, and $R_3 - 2 = 80 - 2 = 78$ inches, respectively. The length of the path is then $\pi(98 + 62 + 78) = 238\pi$ inches.



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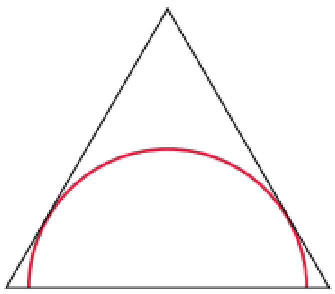
2016 Q25

25. A semicircle is inscribed in an isosceles triangle with base 16 and height 15 so that the diameter of the semicircle is contained in the base of the triangle as shown. What is the radius of the semicircle?



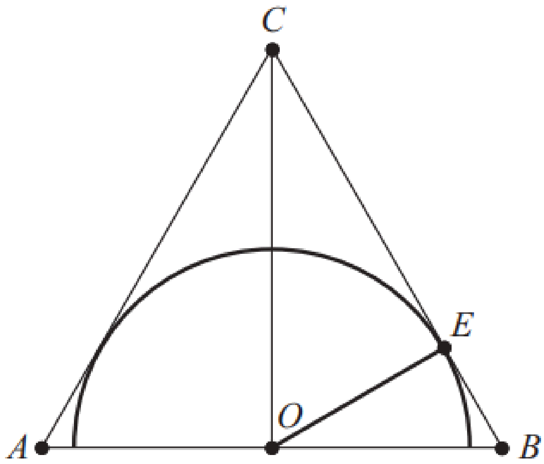
(A) $4\sqrt{3}$ **(B)** $\frac{120}{17}$ **(C)** 10 **(D)** $\frac{17\sqrt{2}}{2}$ **(E)** $\frac{17\sqrt{3}}{2}$

25. Answer (B):

Let O be the midpoint of base $\overline{AB}$ of $\triangle ABC$ and the center of the semicircle. Triangle $\triangle OBC$ is a right triangle with $OB = 8$ and $OC = 15$, and so, by the Pythagorean Theorem, $BC = 17$. Let E be the point where the semicircle intersects $\overline{BC}$, so radius $\overline{OE}$ is perpendicular to $\overline{BC}$. Then $\triangle OEB$ and $\triangle COB$ are similar, and therefore, $OE : CO = OB : CB$. Hence, $\frac{OE}{15} = \frac{8}{17}$ and so $OE = \frac{120}{17}$.

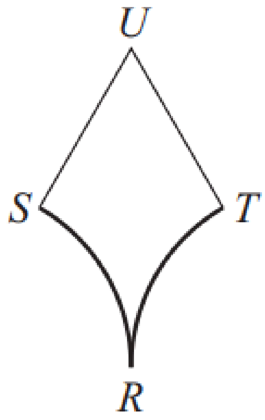
OR

Let O be the center of the semicircle, which is also the midpoint of base $\overline{AB}$. Since $OB = 8$ and $OC = 15$, then by the Pythagorean Theorem $BC = 17$. Let E be the point where the semicircle intersects $\overline{BC}$, so radius $\overline{OE}$ is perpendicular to $\overline{BC}$. Since the area of $\triangle OBC$ is $\frac{1}{2}(BC)(OE) = \frac{1}{2}(OB)(OC)$, then $\frac{1}{2}(17)(OE) = \frac{1}{2}(8)(15)$ and so $OE = 120/17$.



2017 Q25

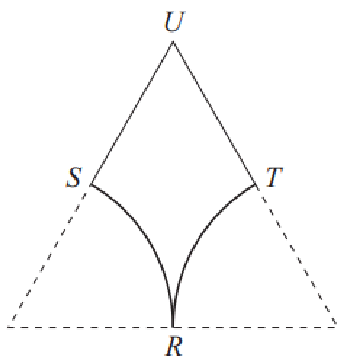
25. In the figure shown, $\overline{US}$ and $\overline{UT}$ are line segments each of length 2, and $m\angle TUS = 60°$. Arcs $\overset{\frown}{TR}$ and $\overset{\frown}{SR}$ are each one-sixth of a circle with radius 2. What is the area of the region shown?



(A) $3\sqrt{3}-\pi$ **(B)** $4\sqrt{3}-\frac{4\pi}{3}$ **(C)** $2\sqrt{3}$ **(D)** $4\sqrt{3}-\frac{2\pi}{3}$

(E) $4+\frac{4\pi}{3}$

25. **Answer (B):** The region shown is what remains when two one-sixth sectors of a circle of radius 2 are removed from an equilateral triangle with side length 4.



The area of an equaliateral triangle with side length s is $\frac{\sqrt{3}}{4}s^2$. Thus the area of the region is $4\sqrt{3}-2\left(\frac{1}{6}\cdot 4\pi\right)=4\sqrt{3}-\frac{4\pi}{3}$.