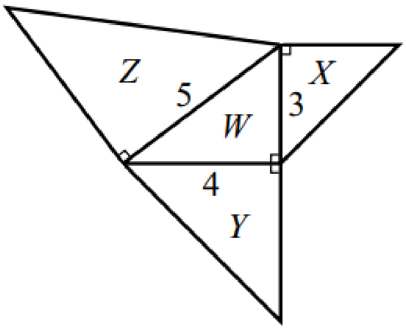
2002 Q16

16. Right isosceles triangles are constructed on the sides of a 3-4-5 right triangle, as shown. A capital letter represents the area of each triangle. Which one of the following is true?



(A) $X + Z = W + Y$ **(B)** $W + X = Z$ **(C)** $3X + 4Y = 5Z$
(D) $X + W = \frac{1}{2}(Y + Z)$ **(E)** $X + Y = Z$

16. **(E)** The areas are $W = \frac{1}{2}(3)(4) = 6$, $X = \frac{1}{2}(3)(3) = 4\frac{1}{2}$, $Y = \frac{1}{2}(4)(4) = 8$ and $Z = \frac{1}{2}(5)(5) = 12\frac{1}{2}$. Therefore, (E) is correct. $X + Y = 4\frac{1}{2} + 8 = 12\frac{1}{2} = Z$. The other choices are incorrect.

OR

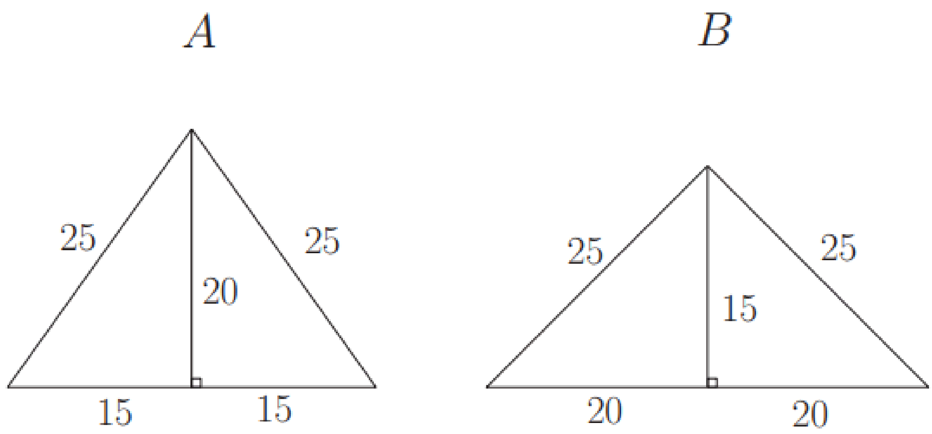
By the Pythagorean Theorem, if squares are constructed on each side of any right triangle, the sum of the areas of the squares on the legs equal the area of the square on the hypotenuse. So $2X + 2Y = 2Z$, and $X + Y = Z$.

2011 Q16

16. Let A be the area of a triangle with sides of length 25, 25, and 30. Let B be the area of a triangle with sides of length 25, 25, and 40. What is the relationship between A and B?

(A) $A = \frac{9}{16}B$ **(B)** $A = \frac{3}{4}B$ **(C)** $A = B$ **(D)** $A = \frac{4}{3}B$ **(E)** $A = \frac{16}{9}B$

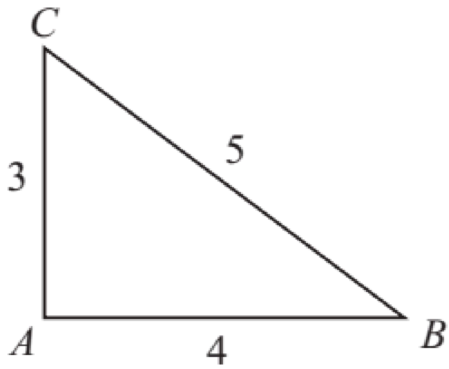
16. **Answer (C):**



The altitude shown divides each triangle into two congruent right triangles. The hypotenuse of each right triangle is 25. In $\triangle A$ the horizontal leg of each right triangle is 15, so the vertical leg is $\sqrt{25^2 - 15^2} = 20$. In $\triangle B$ the horizontal leg of each right triangle is 20, so the vertical leg is 15. The area of $\triangle A$ is $\frac{1}{2}(30)(20) = 300$, and the area of $\triangle B$ is $\frac{1}{2}(40)(15) = 300$, so the two areas are equal.

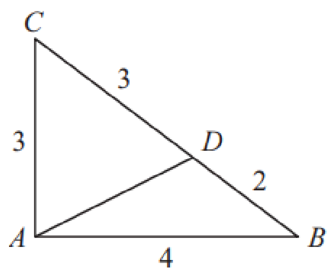
2017 Q16

16. In the figure shown below, choose point D on side $\overline{BC}$ so that $\triangle ACD$ and $\triangle ABD$ have equal perimeters. What is the area of $\triangle ABD$?



(A) $\frac{3}{4}$ **(B)** $\frac{3}{2}$ **(C)** 2 **(D)** $\frac{12}{5}$ **(E)** $\frac{5}{2}$

16. **Answer (D):** Because the perimeters of $\triangle ADC$ and $\triangle ADB$ are equal, $CD = 3$ and $BD = 2$.

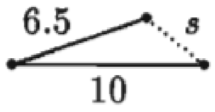


$\triangle ADC$ and $\triangle ADB$ have the same altitude from A, so the area of $\triangle ADC$ will be $3/5$ of the area of $\triangle ABC$, and $\triangle ADB$ will be $\frac{2}{5}$ of the area of $\triangle ABC$. The area of $\triangle ABC$ is $\frac{1}{2} \cdot 3 \cdot 4 = 6$, so the area of $\triangle ADB$ is $\frac{2}{5} \cdot 6 = 12/5$.

1992 Q17

17. The sides of a triangle have lengths $6.5, 10$, and s, where s is a whole number. What is the smallest possible value of s?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7



17. **(B)** For any triangle, the sum of the lengths of any two sides must be greater than the length of the third side. Thus $6.5 + s$ must be greater than 10. The smallest such whole number for s is 4.

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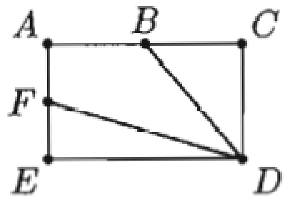
1993 Q18

18. The rectangle shown has length $AC = 32$, width $AE = 20$, and B and F are midpoints of $\overline{AC}$ and $\overline{AE}$, respectively. The area of the quadrilateral $ABDF$ is

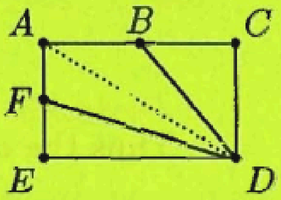
(A) 320 (B) 325 (C) 330
(D) 335 (E) 340



18. **(A)** Rectangle $ACDE$ has area $32 \times 20 = 640$. Triangle BCD has area $(16 \times 20)/2 = 160$, and triangle DEF has area $(10 \times 32)/2 = 160$. The remaining area, $ABDF$, is $640 - (160 + 160) = 320$.

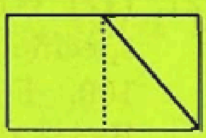
OR

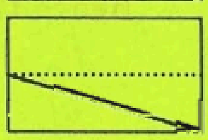
Insert diagonal $\overline{AD}$. The areas of triangles ABD and BCD are $(AB)(CD)/2$ and $(BC)(CD)/2$ which are equal since $AB = BC$. Hence half the area in the rectangle above $\overline{AD}$ is in $ABDF$. Similarly, triangles ADF and DEF have equal areas, and half the area in the rectangle below $\overline{AD}$ is in $ABDF$. Thus, the area of $ABDF$ is $(32 \times 20)/2 = 320$.



OR

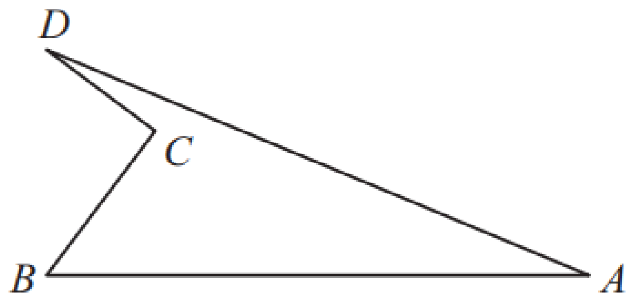
Draw a perpendicular from point B to $\overline{ED}$ to show that the area of $\triangle BCD$ is one fourth of the area of $ACDE$. Similarly, draw a perpendicular from point F to $\overline{CD}$ to show that the area of $\triangle DEF$ is one fourth of the area of $ACDE$. Thus the area of $ABDF$ is one half of the area of $ACDE$, or $(32 \times 20)/2 = 320$.





2017 Q18

18. In the non-convex quadrilateral $ABCD$ shown below, $\angle BCD$ is a right angle, $AB = 12$, $BC = 4$, $CD = 3$, and $AD = 13$.



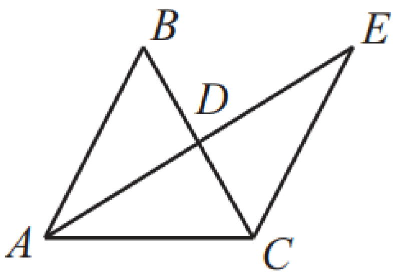
What is the area of quadrilateral $ABCD$?

(A) 12 **(B)** 24 **(C)** 26 **(D)** 30 **(E)** 36

18. **Answer (B):** In right triangle BCD, $3^2 + 4^2 = 5^2$, so $BD = 5$. In $\triangle ABD$, $13^2 = 12^2 + 5^2$, so $\triangle ABD$ is a right triangle with right angle $\angle ABD$. The area of $\triangle ABD$ is $\frac{1}{2} \cdot 5 \cdot 12 = 30$. The area of $\triangle BCD$ is $\frac{1}{2} \cdot 3 \cdot 4 = 6$. So the area of the quadrilateral is $30 - 6 = 24$.

2006 Q19

19. Triangle ABC is an isosceles triangle with $AB = BC$. Point D is the midpoint of both $\overline{BC}$ and $\overline{AE}$, and $\overline{CE}$ is 11 units long. Triangle ABD is congruent to triangle ECD. What is the length of $\overline{BD}$?



(A) 4 **(B)** 4.5 **(C)** 5 **(D)** 5.5 **(E)** 6

19. **(D)** Because triangles ABD and ECD are congruent and triangle ABC is isosceles, $EC = AB = BC = 11$. That means $BD = \frac{11}{2}$ or 5.5.

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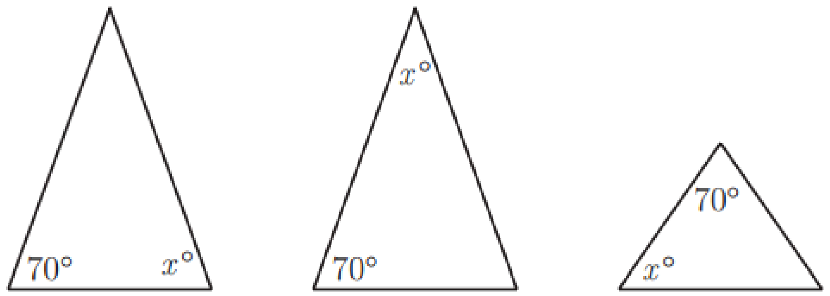
2009 Q19

19. Two angles of an isosceles triangle measure 70° and x°. What is the sum of the three possible values of x?

(A) 95 **(B)** 125 **(C)** 140 **(D)** 165 **(E)** 180

19. **Answer (D):** The two angles measuring $70°$ and $x°$, in an isosceles triangle, could be positioned in three ways, as shown.



If $70°$ and $x°$ are the degree measures of the congruent angles, then $x = 70$. If x is the degree measure of the vertex, then x is $180 - 70 - 70 = 40$. If x is the degree measure of one of the base angles, but not 70, then x is $\frac{1}{2}(180 - 70) = 55$.

The possible values of x are 70, 40 and 55. The sum of these values is $70 + 40 + 55 = 165$.

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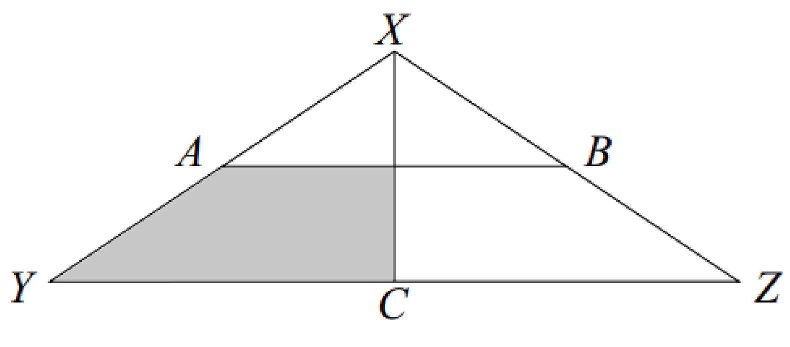
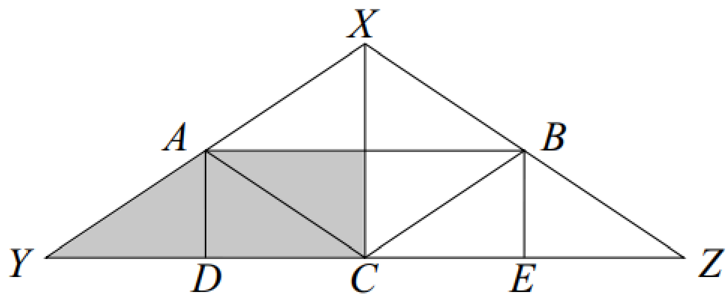
2002 Q20

20. The area of triangle XYZ is 8 square inches. Points A and B are midpoints of congruent segments $\overline{XY}$ and $\overline{XZ}$. Altitude $\overline{XC}$ bisects $\overline{YZ}$. The area (in square inches) of the shaded region is



(A) $1\frac{1}{2}$ **(B)** 2 **(C)** $2\frac{1}{2}$ **(D)** 3 **(E)** $3\frac{1}{2}$

20. **(D)** Segments $\overline{AD}$ and $\overline{BE}$ are drawn perpendicular to $\overline{YZ}$. Segments $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ divide $\triangle XYZ$ into four congruent triangles. Vertical line segments $\overline{AD}$, $\overline{XC}$ and $\overline{BE}$ divide each of these in half. Three of the eight small triangles are shaded, or $\frac{3}{8}$ of $\triangle XYZ$. The shaded area is $\frac{3}{8}(8) = 3$.



OR

Segments $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ divide $\triangle XYZ$ into four congruent triangles, so the area of $\triangle XAB$ is one-fourth the area of $\triangle XYZ$. That makes the area of trapezoid $ABZY$ three-fourths the area of $\triangle XYZ$. The shaded area is one-half the area of trapezoid $ABZY$, or three-eighths the area of $\triangle XYZ$, and $\frac{3}{8}(8) = 3$.