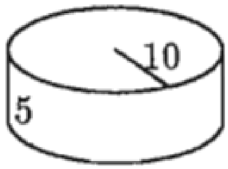
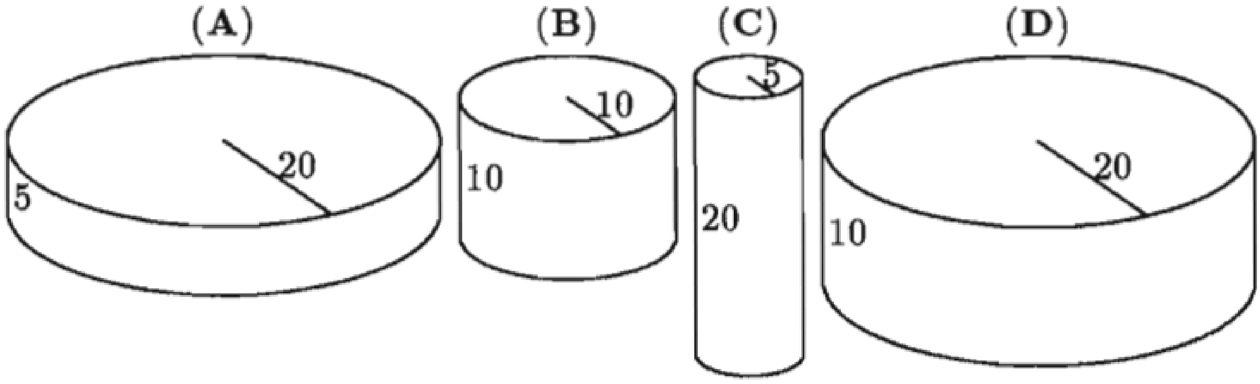
1992 Q16

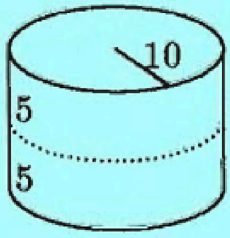
16. Which cylinder has twice the volume of the cylinder shown to the right?





(E) None of the above

16. **(B)** Cylinder **(B)** can be obtained by stacking one copy of the given cylinder on top of another. The formula for the volume of a cylinder with radius r and height h is $V = \pi r^2 h$. Use this to show that none of the other cylinders has twice the volume of the given cylinder:

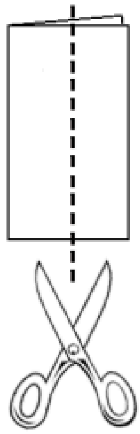


Cylinder	Volume
Given :	$\pi \times 10^2 \times 5 = 500\pi$
(A) :	$\pi \times 20^2 \times 5 = 2000\pi$
(C) :	$\pi \times 5^2 \times 20 = 500\pi$
(D) :	$\pi \times 20^2 \times 10 = 4000\pi$

Note. If the radius remains the same and the height is doubled, then the volume will double, as in **(B)**. Doubling the radius while the height remains the same will multiply the volume by 4, as in **(A)**.

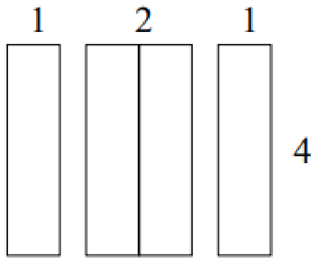
2001 Q16

16. A square piece of paper, 4 inches on a side, is folded in half vertically. Both layers are then cut in half parallel to the fold. Three new rectangles are formed, a large one and two small ones. What is the ratio of the perimeter of one of the small rectangles to the perimeter of the large rectangle?



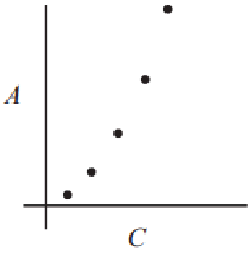
(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{4}{5}$ (E) $\frac{5}{6}$

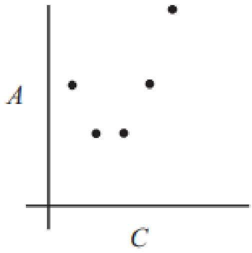
16. (E) The dimensions of the new rectangles are shown. The perimeter of a small rectangle is $4+1+4+1=10$ inches and for the large one it is $4+2+4+2=12$ inches. The ratio is $10/12=5/6$.

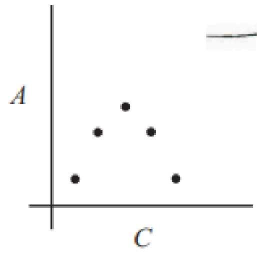


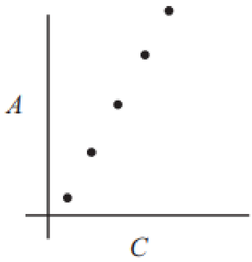
2007 Q16

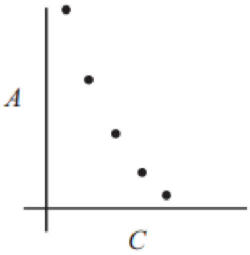
16. Amanda Reckonwith draws five circles with radii 1, 2, 3, 4 and 5. Then for each circle she plots the point (C, A), where C is its circumference and A is its area. Which of the following could be her graph?

(A)


(B)


(C)


(D)


(E)


16. **(A)** The circumferences of circles with radii 1 through 5 are 2π, 4π, 6π, 8π and 10π, respectively. Their areas are, respectively, $\pi, 4\pi$, 9π, 16π and 25π. The points $(2\pi, \pi)$, $(4\pi, 4\pi)$, $(6\pi, 9\pi)$, $(8\pi, 16\pi)$ and $(10\pi, 25\pi)$ are graphed in **(A)**. It is the only graph of an increasing quadratic function, called a parabola.

1994 Q16

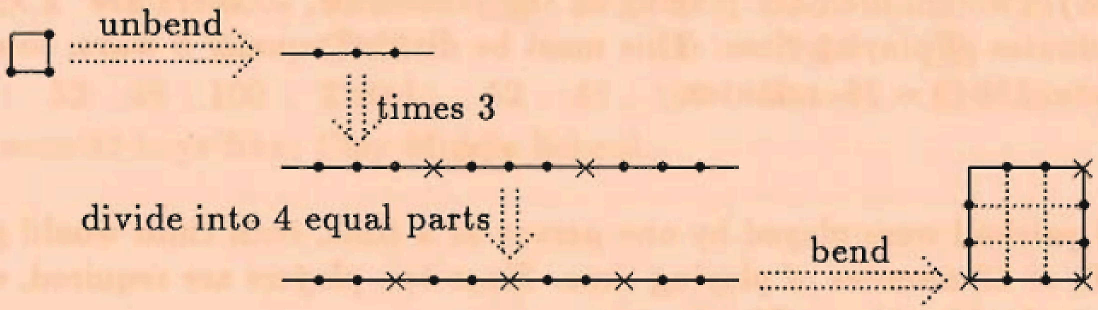
16. The perimeter of one square is 3 times the perimeter of another square. The area of the larger square is how many times the area of the smaller square?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 9

16. **(E)** The perimeter being 3 times larger implies that a side of the larger square is 3 times a side of the smaller square. Thus, since the area of a square is the length of the side squared, it follows that the area of the larger square will be $3^2 = 9$ times the area of the smaller square.

OR

The sketch shows that when the perimeter is tripled, it encloses 9 squares equal to the original square:

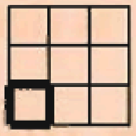


OR

Since the ratio of areas of similar figures is the square of the ratio of any matching linear part, it follows that the ratio of the areas is $\left(\frac{3}{1}\right)^2 = \frac{9}{1}$.

OR

Use a sample case: Area of a 1 by 1 square is 1 square unit.
Area of a 3 by 3 square is 9 square units.



2000 Q16

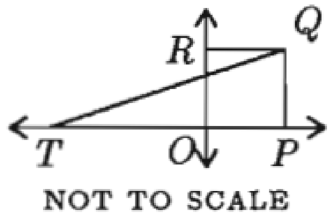
16. In order for Mateen to walk a kilometer(1000m) in his rectangular backyard, he must walk the length 25 times or walk its perimeter 10 times. What is the area of Mateen's backyard in square meters?

(A) 40 **(B)** 200 **(C)** 400 **(D)** 500 **(E)** 1000

16. **Answer (C):** The perimeter is $1000 \div 10 = 100$, and this is two lengths and two widths. The length of the backyard is $1000 \div 25 = 40$. Since two lengths total 80, the two widths total 20, and the width is 10. The area is $10 \times 40 = 400$.

1996 Q17

17. Figure $OPQR$ is a square. Point O is the origin, and point Q has coordinates $(2,2)$. What are the coordinates for T so that the area of triangle PQT equals the area of square $OPQR$?



(A) $(-6,0)$ **(B)** $(-4,0)$ **(C)** $(-2,0)$
(D) $(2,0)$ **(E)** $(4,0)$

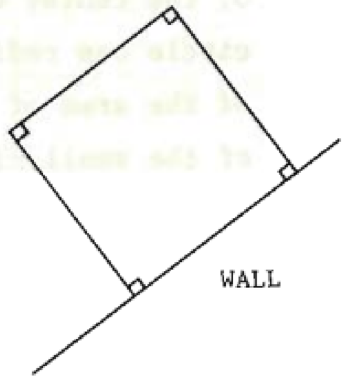
17. **(C)** Since $OPQR$ is a square and point Q has coordinates $(2,2)$, it follows that point P has coordinates $(2,0)$ and point R has coordinates $(0,2)$. Thus the side of square $OPQR$ is 2 and the area of the 2 by 2 square $OPQR$ is $2^2 = 4$. The area of triangle PQT is $(1/2)(PT)(PQ) = (1/2)(PT)(2) = PT$. Since the area of the triangle equals the area of the square, $PT = 4$. Thus the point T has coordinates $(-2,0)$.

OR

If the area of square $OPQR$ and triangle PQT are equal, then the area of the small triangle with side $\overline{RQ}$ that is in the square but not in the triangle must equal the area of the small triangle with side $\overline{OT}$ that is in the triangle but not in the square. These two triangles will be the same size and shape when $\overline{QT}$ intersects $\overline{RO}$ at its midpoint and $OT = RQ = 2$, so T must have coordinates $(-2,0)$.

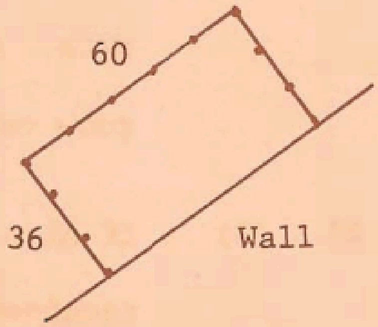
1986 Q18

18. A rectangular grazing area is to be fenced off on three sides using part of a 100 meter rock wall as the fourth side. Fence posts are to be placed every 12 meters along the fence including the two posts where the fence meets the rock wall. What is the fewest number of posts required to fence an area 36 m by 60 m?



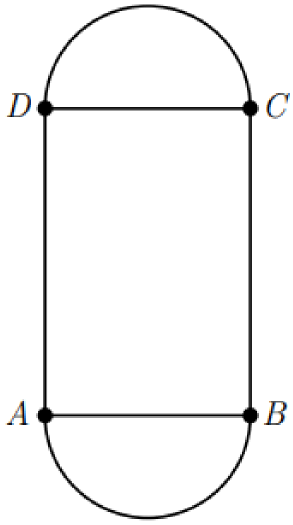
A) 11 B) 12 C) 13 D) 14 E) 16

18. (B) The fewest number of posts is used if the wall serves as the longer side of the rectangular grazing area. Thus there are 6 posts on the 60 meter side (including the corners) and 3 more posts on each 36 meter side for a total of 12 posts.



2010 Q18

18. A decorative window is made up of a rectangle with semicircles on either end. The ratio of AD to AB is $3:2$ and $AB = 30$ inches. What is the ratio of the area of the rectangle to the combined areas of the semicircles?

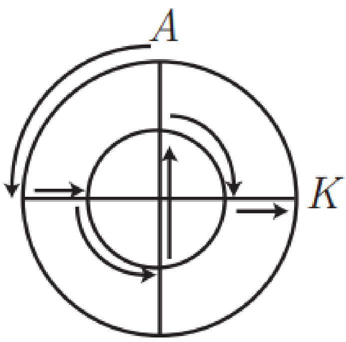


(A) $2:3$ **(B)** $3:2$ **(C)** $6:\pi$ **(D)** $9:\pi$ **(E)** $30:\pi$

18. **Answer (C):** The given ratio implies that $AD = \frac{3}{2}AB = \frac{3}{2} \cdot 30 = 45$ inches, so the area of the rectangle is $AB \cdot AD = 30 \cdot 45$ square inches. The 2 semicircles make 1 circle with radius $= 15$ inches. The area of the circle is $15^2\pi$ square inches. The ratio of the areas is $\frac{30 \cdot 45}{15 \cdot 15\pi} = \frac{6}{\pi}$.

2008 Q18

18. Two circles that share the same center have radii 10 meters and 20 meters. An aardvark runs along the path shown, starting at A and ending at K. How many meters does the aardvark run?

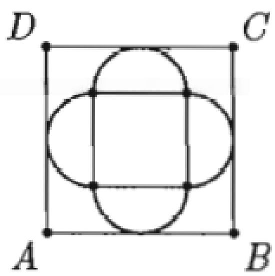


(A) $10\pi + 20$ **(B)** $10\pi + 30$ **(C)** $10\pi + 40$ **(D)** $20\pi + 20$ **(E)** $20\pi + 40$

18. **Answer (E):** The length of first leg of the aardvark's trip is $\frac{1}{4}(2\pi \times 20) = 10\pi$ meters. The third and fifth legs are each $\frac{1}{4}(2\pi \times 10) = 5\pi$ meters long. The second and sixth legs are each 10 meters long, and the length of the fourth leg is 20 meters. The length of the total trip is $10\pi + 5\pi + 5\pi + 10 + 10 + 20 = 20\pi + 40$ meters.

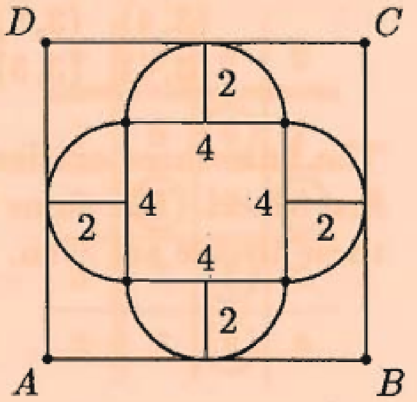
1994 Q19

19. Around the outside of a 4 by 4 square, construct four semicircles (as shown in the figure) with the four sides of the square as their diameters. Another square, $ABCD$, has its sides parallel to the corresponding sides of the original square, and each side of $ABCD$ is tangent to one of the semicircles. The area of the square $ABCD$ is



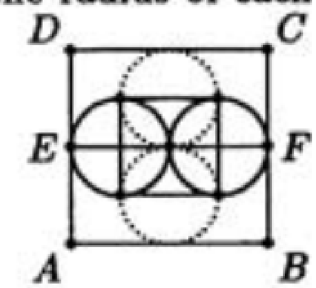
(A) 16 **(B)** 32 **(C)** 36 **(D)** 48 **(E)** 64

19. **(E)** The radius of each semicircle is 2, since it is 1/2 the length of a side of the 4 by 4 square. Since the length of a side of $ABCD$ is the length of a side of the 4 by 4 square plus two radii of semicircles [see figure], each side of $ABCD$ measures $4 + 2(2) = 8$, so the area of $ABCD$ is $8^2 = 64$.



OR

Complete each semicircle to a circle, and note that since the radius of each circle is half the side-length of the smaller square, the four circles must intersect at the center of both squares. Thus, $AB = EF = 8$ since the length of $\overline{EF}$ is the length of two diameters. Hence, the area of $ABCD$ is $8^2 = 64$.



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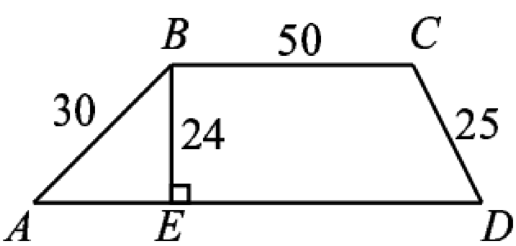
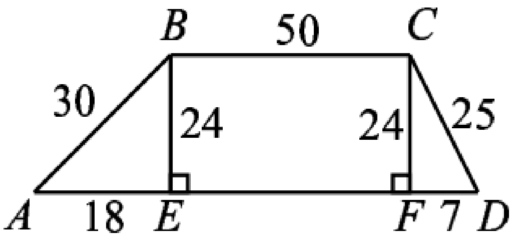
2005 Q19

19. What is the perimeter of trapezoid $ABCD$?



(A) 180 **(B)** 188 **(C)** 196 **(D)** 200 **(E)** 204

19. **(A)**

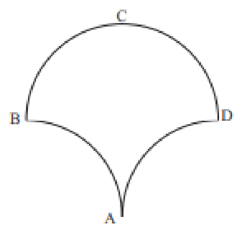


By the Pythagorean Theorem, $AE = \sqrt{30^2 - 24^2} = \sqrt{324} = 18$. (Or note that triangle AEB is similar to a 3-4-5 right triangle, so $AE = 3 \times 6 = 18$.)

Also $CF = 24$ and $FD = \sqrt{25^2 - 24^2} = \sqrt{49} = 7$. The perimeter of the trapezoid is $50 + 30 + 18 + 50 + 7 + 25 = 180$.

2000 Q19

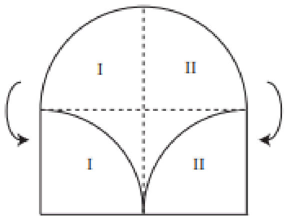
19. Three circular arcs of radius 5 units bound the region shown. Arcs AB and AD are quarter-circles, and arc BCD is a semi-circle. What is the area, in square units, of the region?



(A) 25 **(B)** $10 + 5\pi$ **(C)** 50 **(D)** $50 + 5\pi$

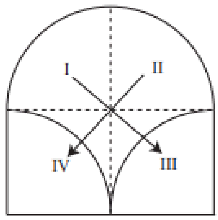
(E) 25π

19. **Answer (C):** Divide the semicircle in half and rotate each half down to fill the space below the quarter-circles. The figure formed is a rectangle with dimensions 5 and 10. The area is 50.



OR

Slide I into III and II into IV as indicated by the arrows to create the 5×10 rectangle.



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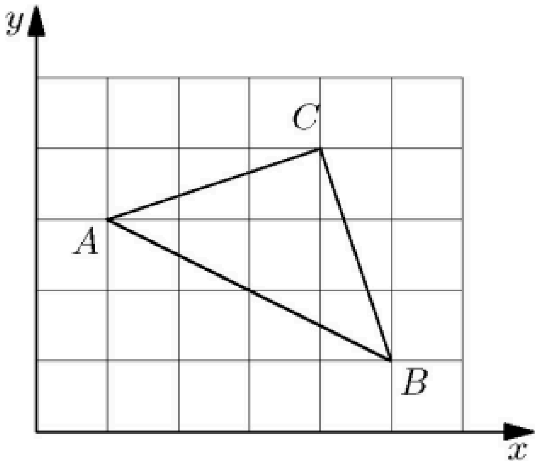
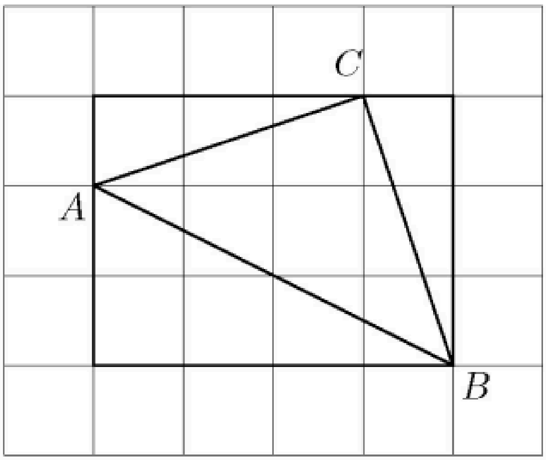
2015 Q19

19. A triangle with vertices at $A = (1, 3)$, $B = (5, 1)$, and $C = (4, 4)$ is plotted on a 6×5 grid. What fraction of the grid is covered by the triangle?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$



19. **Answer (A):** The triangle is inscribed in a 4×3 rectangle with vertices at $(1,1)$, $(1,4)$, $(5,4)$, and $(5,1)$. Three triangular regions are inside the 4×3 rectangle but outside $\triangle ABC$. The area of the lower-left triangle is $\frac{1}{2} \cdot 4 \cdot 2 = 4$ square units. The area of the upper-left triangle is $\frac{1}{2} \cdot 1 \cdot 3 = \frac{3}{2}$ square units. The area of the third triangle is also $\frac{1}{2} \cdot 1 \cdot 3 = \frac{3}{2}$ square units. So the area of $\triangle ABC$ is $12 - 4 - \frac{3}{2} - \frac{3}{2} = 5$ square units. The area of the 6×5 grid is 30 square units. Thus, the fraction covered by the triangle is $\frac{5}{30} = \frac{1}{6}$.

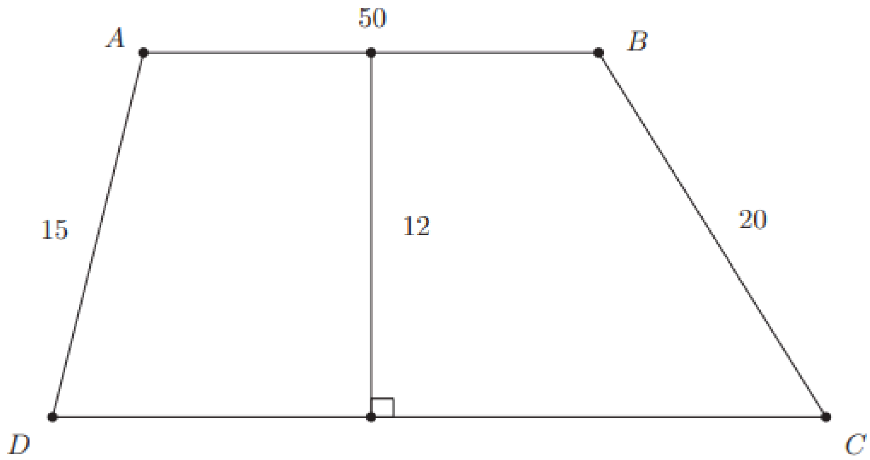


OR

Pick's Theorem says that the area of a polygonal region whose vertices are at lattice points (points whose coordinates are integers) is given by $A = I + \frac{1}{2}B - 1$ where I is the number of lattice points in the interior of the region and B is the number of lattice points on the boundary. Referring to the figure above, there are $B = 4$ lattice points on the boundary of $\triangle ABC$ at $(1,3)$, $(3,2)$, $(5,1)$, and $(4,4)$. There are $I = 4$ points with integer coordinates in the interior of $\triangle ABC$ at $(2,3)$, $(3,3)$, $(4,2)$, and $(4,3)$. Then the area of $\triangle ABC$ is $I + \frac{1}{2}B - 1 = 4 + 2 - 1 = 5$ square units. As before, the fraction of the rectangle covered by the triangle is $\frac{5}{30} = \frac{1}{6}$.

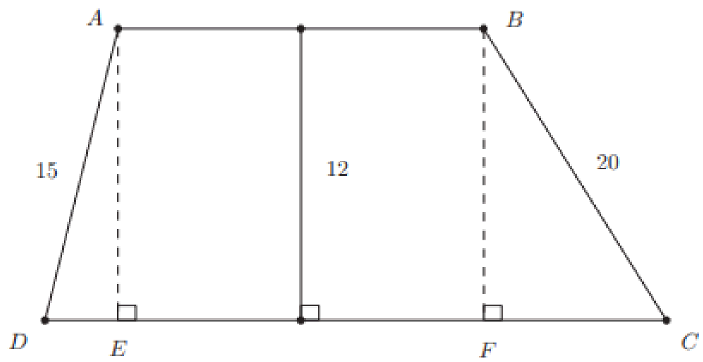
2011 Q20

20. Quadrilateral $ABCD$ is a trapezoid, $AD = 15$, $AB = 50$, $BC = 20$, and the altitude is 12. What is the area of the trapezoid?



(A) 600 **(B)** 650 **(C)** 700 **(D)** 750 **(E)** 800

20. **Answer (D):** Let E and F be the feet of the perpendiculars from A and B to $\overline{DC}$. In right $\triangle AED$, $DE^2 = 15^2 - 12^2 = 225 - 144 = 81$, so $DE = 9$. In right $\triangle BFC$, $FC^2 = 20^2 - 12^2 = 400 - 144 = 256$, so $FC = 16$.



Right $\triangle AED$ has area $\frac{1}{2} \cdot 9 \cdot 12 = 54$, right $\triangle BFC$ has area $\frac{1}{2} \cdot 16 \cdot 12 = 96$, and rectangle $ABFE$ has area $50 \cdot 12 = 600$. The trapezoid $ABCD$ has area $54 + 96 + 600 = 750$.

OR

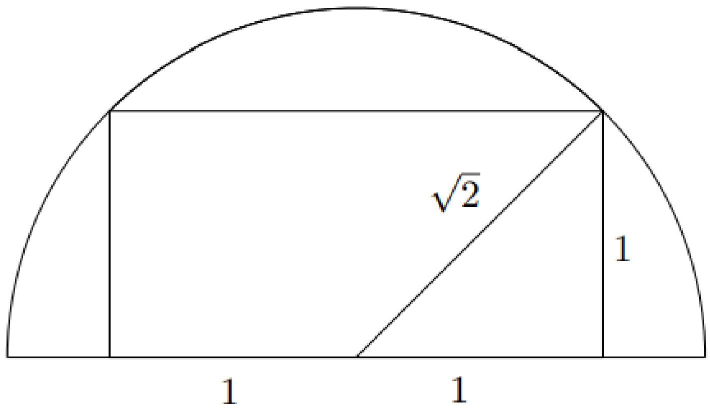
Begin as in the first solution and note that $DC = DE + EF + FC = 9 + 50 + 16 = 75$. Then the area of trapezoid is $\frac{1}{2}(AB + DC) \cdot AE = \frac{1}{2}(50 + 75) \cdot 12 = 125 \cdot 6 = 750$.

2013 Q20

20. A 1×2 rectangle is inscribed in a semicircle with the longer side on the diameter. What is the area of the semicircle?

(A) $\frac{\pi}{2}$ **(B)** $\frac{2\pi}{3}$ **(C)** π **(D)** $\frac{4\pi}{3}$ **(E)** $\frac{5\pi}{3}$

20. **Answer (C):**

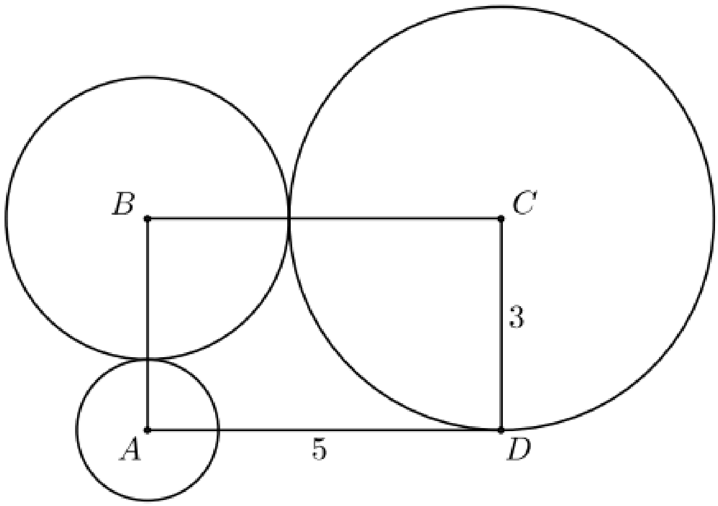


By the Pythagorean Theorem, the radius of the semicircle is $\sqrt{2}$, so its area is $\frac{\pi(\sqrt{2})^2}{2} = \pi$.

2014 Q20

20. Rectangle $ABCD$ has sides $CD = 3$ and $DA = 5$. A circle of radius 1 is centered at A, a circle of radius 2 is centered at B, and a circle of radius 3 is centered at C. Which of the following is closest to the area of the region inside the rectangle but outside all three circles?

(A) 3.5 **(B)** 4.0 **(C)** 4.5 **(D)** 5.0 **(E)** 5.5



20. **Answer (B):** The areas of the quarter-circles are $\frac{\pi}{4}$, π and $\frac{9\pi}{4}$. Their total area is $\frac{7\pi}{2}$. Using $\frac{22}{7}$ as an approximation of π, this is $\frac{7}{2} \cdot \frac{22}{7} = 11$, leaving $15 - 11 = 4$ for the desired area. (Using 3.14 for π yields 4.01.)