1 / 8

1997 Q23

23. There are positive integers that have these properties:

I. the sum of the squares of their digits is 50, and

II. each digit is larger than the one to its left.

The product of the digits of the largest integer with both properties is

(A) 7 (B) 25 (C) 36 (D) 48 (E) 60

23. **(C)** To meet the first condition, numbers which sum to 50 must be chosen from the set of squares {1, 4, 9, 16, 25, 36, 49}. To meet the second condition, the squares selected must be different. Consequently, there are three possibilities: $1 + 49$, $1 + 4 + 9 + 36$, and $9 + 16 + 25$. These correspond to the integers 17, 1236, and 345, respectively. The largest is 1236, and the product of its digits is $1 \cdot 2 \cdot 3 \cdot 6 = 36$.

2 / 8

2010 Q24

24. What is the correct ordering of the three numbers 10^8, 5^{12}, and 2^{24} ?

(A) $2^{24} < 10^8 < 5^{12}$ **(B)** $2^{24} < 5^{12} < 10^8$ **(C)** $5^{12} < 2^{24} < 10^8$

(D) $10^8 < 5^{12} < 2^{24}$ **(E)** $10^8 < 2^{24} < 5^{12}$

24. **Answer (A):** $2^{24} = (2^8) \cdot (2^{16}) = (2^8) \cdot (4^8) < (2^8) \cdot (5^8) = 10^8 = (4^4) \cdot (5^8) < (5^4) \cdot (5^8) = 5^{12}$.

2011 Q24

24. In how many ways can 10,001 be written as the sum of two primes?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

24. **Answer (A):** If the sum of two numbers is odd, one number must be even and the other number must be odd. Because all primes except 2 are odd, 2 must be one of the summands. Because $10{,}001 = 2 + 9999$, and $9999 = 9 \cdot 1111$ is not prime, there are no solutions.

2015 Q24

24. A baseball league consists of two four-team divisions. Each team plays every other team in its division N games. Each team plays every team in the other division M games with $N > 2M$ and $M > 4$. Each team plays a 76 game schedule. How many games does a team play within its own division?

(A) 36 **(B)** 48 **(C)** 54 **(D)** 60 **(E)** 72

24. **Answer (B):** The number of games played by a team is $3N + 4M = 76$. Because $M > 4$ and $N > 2M$ it follows that $N > 8$. Because 4 divides both 76 and $4M$, 4 must divide $3N$ and hence N. If $N = 12$ then $M = 10$ and the condition $N > 2M$ is not satisfied. If $N \geq 20$ then $M \leq 4$ and the condition $M > 4$ is not satisfied. So the only possibility is $N = 16$ and $M = 7$. So each team plays $3 \cdot 16 = 48$ games within its division and $4 \cdot 7 = 28$ games against the other division.

OR

The total number of games played by each team is $3N + 4M = 76$. Make a chart of possibilities with $M > 4$:

M	$4M$	$76 - 4M = 3N$	N
5	20	56	(not an integer)
6	24	52	(not an integer)
7	28	48	16
8	32	44	(not an integer)
9	36	40	(not an integer)
10	40	36	12
11	44	32	(not an integer)
12	48	28	(not an integer)
13	52	24	8

Only $M = 7$ and $N = 16$ satisfy the conditions.

The case $M = 10$ and $N = 12$ violates the condition $N > 2M$.

So each team plays $3N = 48$ divisional games and $4M = 28$ games against the other division.

This is modeled on the Pioneer Baseball League with teams in Colorado, Idaho, Montana, and Utah.

2016 Q24

24. The digits 1, 2, 3, 4, and 5 are each used once to write a five-digit number $PQRST$. The three-digit number PQR is divisible by 4, the three-digit number QRS is divisible by 5, and the three-digit number RST is divisible by 3. What is P?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

24. Answer (A):

Since QRS is divisible by 5, we know that $S = 5$. Since PQR is divisible by 4, we know that QR is 12, 32, or 24. So RST will be either $25T$ or $45T$ and divisible by 3. Using the available digits, 453 is the only number that is divisible by 3. So $T = 3$, $R = 4$, and $P = 1$.

6 / 8

1994 Q25

25. Find the sum of the digits in the answer to

$$\underbrace{9999\cdots99}_{94\text{ nines}} \times \underbrace{4444\cdots44}_{94\text{ fours}}$$

where a string of 94 nines is multiplied by a string of 94 fours.

(A) 846 (B) 855 (C) 945 (D) 954 (E) 1072

25. **(A)** Since $\underbrace{9999\cdots99}_{94\text{ nines}} = 1\underbrace{0000\cdots00}_{94\text{ zeros}} - 1$, we have

$$\underbrace{9999\cdots99}_{94\text{ nines}} \times \underbrace{4444\cdots44}_{94\text{ fours}} = (1\underbrace{0000\cdots00}_{94\text{ zeros}} - 1) \times \underbrace{4444\cdots44}_{94\text{ fours}}$$
$$= (\underbrace{4444\cdots44}_{94\text{ fours}}\underbrace{0000\cdots00}_{94\text{ zeros}} - \underbrace{4444\cdots44}_{94\text{ fours}}$$

which is

$$\begin{array}{r} 4444\cdots44\,0000\cdots00 \\ -\ 4444\cdots44 \\ \hline \underbrace{4444\cdots4}_{93\text{ fours}}3\underbrace{5555\cdots5}_{93\text{ fives}}6 \end{array}$$

The sum of the digits of this answer is

$$93(4) + 3 + 93(5) + 6 = 93(4+5) + (3+6) = 94(9) = 846.$$

OR

Try smaller cases to observe a pattern:

$$\begin{array}{rcll} 9 \times 4 & = & 36 & \text{sum} = 9 = 9 \times 1 \\ 99 \times 44 & = & 4356 & = 18 = 9 \times 2 \\ 999 \times 444 & = & 443556 & = 27 = 9 \times 3 \\ 9999 \times 4444 & = & 44435556 & = 36 = 9 \times 4 \\ \vdots & = & \vdots & \\ \underbrace{9999\cdots99}_{94\text{ nines}} \times \underbrace{4444\cdots44}_{94\text{ fours}} & = & \underbrace{4444\cdots4}_{93\text{ fours}}3\underbrace{5555\cdots5}_{93\text{ fives}}6 & = 9 \times 94 \end{array}$$

The sum of the digits of this answer is

$$93(4) + 3 + 93(5) + 6 = 93(4+5) + (3+6) = 94(9) = 846.$$

Query. What is the sum of the digits when any 94-digit number is multiplied by $\underbrace{9999\cdots99}_{94\text{ nines}}$?

1997 Q25

25. All of the even numbers from 2 to 98 inclusive, except those ending in 0, are multiplied together. What is the rightmost digit (the units digit) of the product?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

25. **(D)** If the numbers 2, 4, 6, and 8 are multiplied, the product is 384, so 4 is the final digit of the product of a set of numbers ending in 2, 4, 6, and 8. Since there are ten such sets of numbers, the final digit of the overall product is the same as the final digit of 4^{10}. Now, $4^{10} = (4^2)^5 = 16^5$. Next, consider 6^5. Since any number of 6's multiply to give 6 as the final digit, the final digit of the required product is 6.

8 / 8

2001 Q25

25. There are 24 four-digit whole numbers that use each of the four digits 2, 4, 5 and 7 exactly once. Only one of these four-digit numbers is a multiple of another one. Which of the following is it?

(A) 5724 (B) 7245 (C) 7254 (D) 7425 (E) 7542

25. (D) Six of the 24 numbers are in the 2000s, six in the 4000s, six in the 5000s and six in the 7000s. Doubling and tripling numbers in the 2000s produce possible solutions, but any multiple of those in the other sets is larger than 8000.

Units digits of the numbers are 2, 4, 5 and 7, so their doubles will end in 4, 8, 0 and 4, respectively. Choice (A) 5724 ends in 4 but $5724/2 = 2862$, not one of the 24 numbers. Likewise, choice (C) 7254 produces $7254/2 = 3627$, also not one of the numbers. When the units digits are tripled the resulting units digits are 6, 2, 5 and 1 and choices (B) 7245, (D) 7425 and (E) 7542 are possibilities. Division by 3 yields 2415, 2475 and 2514 respectively. Only the second of these numbers is one of the 24 given numbers. Choice (D) is correct.