6-10 NUMBER Number Theory

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6. The smallest product one could obtain by multiplying two numbers in the set { -7, -5, -1, 1, 3 } is

1987 Q6

A) -35 B) -21 C) -15 D) -1 E) 3

6. B A negative product occurs when multiplying a positive number by a negative number. The minimum product will occur, then, when multiplying the smallest negative number by the largest positive number. In this case, that product is (-7)(3) = -21.

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1990 Q7

7. When three different numbers from the set { –3, –2, –1, 4, 5 } are multiplied, the largest possible product is

A) 10 B) 20 C) 30 D) 40 E) 60

7. C For the product of three numbers to be positive, either all three of the numbers must be positive or one must be positive and two must be negative. Since there are only two positive numbers, only the latter case is possible. Thus the largest such product is $(-3)(-2)(5) = 30$.

2016 Q7

7. Which of the following numbers is **not** a perfect square?

(A) 1^{2016} **(B)** 2^{2017} **(C)** 3^{2018} **(D)** 4^{2019} **(E)** 5^{2020}

7. Answer (B):

The numbers 1^{2016}, 3^{2018}, 5^{2020} have even exponents and hence are squares. The number 2^{2017} is not a perfect square because it is twice a square $2(2^{1008})^2$. SInce $4^{2019} = (2^2)^{2019} = 2^{4038}$, it is also a perfect square.

OR

A positive integer power of a square is again a square. This eliminates choices (A) and (D). An even power of any integer is a square. This eliminates choices (C) and (E) The only remaining choice is (B), and in fact, an odd power of a non-square cannot be a square.

2017 Q7

7. Let Z be a 6-digit positive integer, such as 247247, whose first three digits are the same as its last three digits taken in the same order. Which of the following numbers must be a factor of Z ?

(A) 11 **(B)** 19 **(C)** 101 **(D)** 111 **(E)** 1111

7. **Answer (A):** Assume Z has the form $abcabc$. Then

$$Z = 1001 \cdot abc = 7 \cdot 11 \cdot 13 \cdot abc$$

So 11 must be a factor of Z.

OR

A positive integer is divisible by 11 if and only if the difference of the sums of the digits in the even and odd positions in the number is divisible by 11. For $Z = abcabc$ the sum of the digits in the even positions is equal to the sum of the digits in the odd positions, so the difference of the two sums is 0. Hence, 11 divides Z.

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1987 Q8

8.

$$\begin{array}{r} 9876 \\ A32 \\ B1 \\ \hline \end{array}$$

If A and B are nonzero digits, then the number of digits (not necessarily different) in the sum of the three whole numbers is

A) 4 B) 5 C) 6 D) 9 E) depends on the values of A and B

8. B 7 + 3 + B > 9, so we "carry 1" from the ten's column to the hundred's column. Similarly 1 + 8 + A > 9 since A > 0, so we "carry 1" from the hundred's place to the thousand's place. 1 + 9 = 10, so the sum is a 5-digit number of the form 10CD9.

OR

Since A is a digit from 1 to 9, the sum 9876 + A32 must be between 10,000 and 100,000. Thus the sum of the three whole numbers must have 5 digits.

2005 Q8

8. Suppose m and n are positive odd integers. Which of the following must also be an odd integer?

(A) $m+3n$ **(B)** $3m-n$ **(C)** $3m^2+3n^2$ **(D)** $(nm+3)^2$ **(E)** $3mn$

8. **(E)**

To check the possible answers, choose the easiest odd numbers for m and n. If $m = n = 1$, then

$$m + 3n = 4, \quad 3m - n = 2, \quad 3m^2 + 3n^2 = 6, \quad (mn + 3)^2 = 16 \text{ and } 3mn = 3.$$

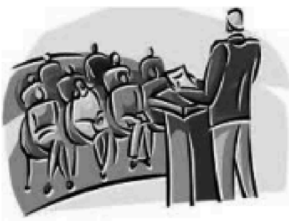
This shows that (A), (B), (C) and (D) can be even when m and n are odd. On the other hand, because the product of odd integers is always odd, $3mn$ is always odd if m and n are odd.

Questions: Which of the expressions are always even if m and n are odd? What are the possibilities if m and n are both even? If one is even and the other odd?

2014 Q8

8. Eleven members of the Middle School Math Club each paid the same amount for a guest speaker to talk about problem solving at their math club meeting. They paid their guest speaker $\$\underline{1}\,\underline{A}\,\underline{2}$. What is the missing digit A of this 3-digit number?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4



8. **Answer (D):** The multiples of 11 between 102 and 192 are 110, 121, 132, 143, 154, 165, 176, and 187. Only 132 satisfies the condition, so $A = 3$.

2017 Q8

8. Malcolm wants to visit Isabella after school today and knows the street where she lives but doesn't know her house number. She tells him, "My house number has two digits, and exactly three of the following four statements about it are true."

(1) It is prime.

(2) It is even.

(3) It is divisible by 7.

(4) One of its digits is 9.

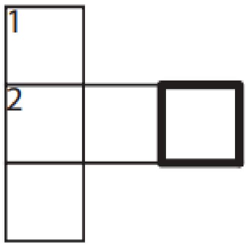
This information allows Malcolm to determine Isabella's house number. What is its units digit?

(A) 4 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

8. **Answer (D):** There are no two-digit even primes, so statements (1) and (2) cannot both be true. Also, no two-digit prime is divisible by 7, so statements (1) and (3) cannot both be true. Because there is only one false statement, it must be (1), so Isabella's house number is an even two-digit multiple of 7 that has a digit of 9. The number is even, so the 9 must be the tens digit. The only even multiple of 7 between 90 and 99 is 98, so the units digit is 8.

2000 Q9

9. Three-digit powers of 2 and 5 are used in this *cross-number* puzzle. What is the only possible digit for the outlined square?



ACROSS	DOWN
2. 2^m	**1.** 5^n

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

9. **Answer (D):** The 3-digit powers of 5 are 125 and 625, so space 2 is filled with a 2. The only 3-digit power of 2 beginning with 2 is 256, so the outlined block is filled with a 6.

2006 Q9

9. What is the product of $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \ldots \times \frac{2006}{2005}$?

(A) 1 **(B)** 1002 **(C)** 1003 **(D)** 2005 **(E)** 2006

9. **(C)** Note that in each fraction, the numerator is the same as the denominator in the next fraction, so they divide. The product of $\frac{\not{3}}{2} \times \frac{\not{4}}{\not{3}} \times \frac{\not{5}}{\not{4}} \times \ldots \times \frac{2006}{\not{2005}} = \frac{2006}{2} = 1003$.

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2016 Q9

9. What is the sum of the distinct prime integer divisors of 2016?

(A) 9 **(B)** 12 **(C)** 16 **(D)** 49 **(E)** 63

9. Answer (B):

The prime factorization of 2016 is: $2016 = (2^5)(3^2)(7)$, so the distinct prime divisors of 2016 are 2, 3, and 7, and their sum is $2+3+7 = 12$.

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2007 Q10

10. For any positive integer n, define $\boxed{n}$ to be the sum of the positive factors of n. For example, $\boxed{6} = 1+2+3+6 = 12$. Find $\boxed{\boxed{11}}$.

(A) 13 **(B)** 20 **(C)** 24 **(D)** 28 **(E)** 30

10. **(D)** First calculate $\boxed{11} = 1 + 11 = 12$. So

$$\boxed{\boxed{11}} = \boxed{12} = 1 + 2 + 3 + 4 + 6 + 12 = 28$$

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2012 Q10

10. How many 4-digit numbers greater than 1000 are there that use the four digits of 2012?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 12

10. **Answer (D):** To form a number greater than 1000, the first digit must be 1 or 2. If the first digit is a 1, the remaining numbers could be 022, 202, or 220. If the first digit is a 2, the remaining numbers could be 012, 021, 102, 120, 201, or 210. So there are $3 + 6 = 9$ ways to form a number greater than 1000.