2007 Q19

19. Pick two consecutive positive integers whose sum is less than 100. Square both of those integers and then find the difference of the squares. Which of the following could be the difference?

(A) 2 **(B)** 64 **(C)** 79 **(D)** 96 **(E)** 131

19. **(C)** One of the squares of two consecutive integers is odd and the other is even, so their difference must be odd. This eliminates A, B and D. The largest consecutive integers that have a sum less than 100 are 49 and 50, whose squares are 2401 and 2500, with a difference of 99. Because the difference of the squares of consecutive positive integers increases as the integers increase, the difference cannot be 131. The difference between the squares of 40 and 39 is 79.

OR

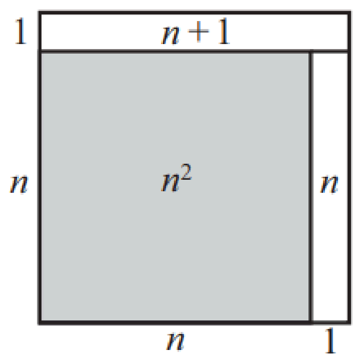
Let the consecutive integers be n and $n+1$, with $n \leq 49$. Then

$$(n+1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1 = n + (n+1).$$

That means the difference of the squares is an odd number. Therefore, the difference is an odd number less than or equal to $49 + (49 + 1) = 99$, and choice C is the only possible answer. The sum of $n = 39$ and $n + 1 = 40$ is 79.

Note: The difference of the squares of any two consecutive integers is not only odd but also the sum of the two consecutive integers. Every positive odd integer greater than 1 and less than 100 could be the answer.

Seen in geometric terms, $(n+1)^2 - n^2$ looks like



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2002 Q19

19. How many whole numbers between 99 and 999 contain exactly one 0?

(A) 72 **(B)** 90 **(C)** 144 **(D)** 162 **(E)** 180

19. **(D)** Numbers with exactly one zero have the form $\_0\_$ or $\_\_0$, where the blanks are not zeros. There are $(9 \cdot 1 \cdot 9) + (9 \cdot 9 \cdot 1) = 81 + 81 = 162$ such numbers.

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2016 Q19

19. The sum of 25 consecutive even integers is 10,000. What is the largest of these 25 consecutive even integers?

(A) 360 **(B)** 388 **(C)** 412 **(D)** 416 **(E)** 424

19. Answer (E):

The average of the 25 even integers is $10000/25 = 400$. So 12 consecutive even integers will be larger than 400 and 12 consecutive even integers will be smaller than 400. The sum $376 + 378 + \cdots + 398 + 400 + 402 + \cdots + 422 + 424 = 10000$. The largest of these numbers is 424.

OR

The average of the 25 even integers is $\frac{10000}{25} = 400$. Since 12 of the consecutive even integers are larger than 400, the largest is $400 + 12 \cdot 2 = 424$.

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2017 Q19

19. For any positive integer M, the notation $M!$ denotes the product of the integers 1 through M. What is the largest integer n for which 5^n is a factor of the sum $98! + 99! + 100!$?

(A) 23 **(B)** 24 **(C)** 25 **(D)** 26 **(E)** 27

19. **Answer (D):** Factoring yields $98!+99!+100! = 98!(1+99+100\cdot 99) = 98!(100+100\cdot 99) = 98!(100)(1+99) = 98!\cdot 100^2$. The exponent of 5 in 98! is $19+3=22$, one for each multiple of 5 and one more for each multiple of 25. Thus the exponent of 5 in the product is $22+4=26$ as $100^2 = 2^4\cdot 5^4$.

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1987 Q20

20. "If a whole number n is not prime, then the whole number n - 2 is not prime." A value of n which shows this statement to be false is

A) 9 B) 12 C) 13 D) 16 E) 23

20. A To show the statement is false, we must find a value of n so that n is not prime and n - 2 is prime. Such a value is n = 9.

1993 Q20

20. When $10^{93} - 93$ is expressed as a single whole number, the sum of the digits is

(A) 10 **(B)** 93 **(C)** 819 **(D)** 826 **(E)** 833

20. **(D)** Since $10^{93} = 1\overbrace{00\cdots00}^{93\text{ zeros}}$, we have

$$\begin{array}{r} 100\cdots000 \\ -\quad 93 \\ \hline \underbrace{99\cdots9}_{91\text{ nines}}07 \end{array}$$

and the sum of the digits is $(91 \times 9) + 7 = 826$.

OR

Look for a pattern using simpler cases:

$$\begin{aligned}
10^2 - 93 &= 100 - 93 &= 07 \\
10^3 - 93 &= 1,000 - 93 &= \underline{9}07 \\
10^4 - 93 &= 10,000 - 93 &= \underline{99}07 \\
10^5 - 93 &= 100,000 - 93 &= \underline{999}07 \\
&\vdots & \vdots \\
10^{93} - 93 &= &= \underbrace{999\ldots9}_{91\text{ nines}}07
\end{aligned}$$

Thus the sum of the digits is $(91 \times 9) + 7 = 826$.

2005 Q20

20. Alice and Bob play a game involving a circle whose circumference is divided by 12 equally-spaced points. The points are numbered clockwise, from 1 to 12. Both start on point 12. Alice moves clockwise and Bob, counterclockwise. In a turn of the game, Alice moves 5 points clockwise and Bob moves 9 points counterclockwise. The game ends when they stop on the same point. How many turns will this take?

(A) 6 **(B)** 8 **(C)** 12 **(D)** 14 **(E)** 24

20. **(A)** Write the points where Alice and Bob will stop after each move and compare points.

Move	0	1	2	3	4	5	6
Alice:	12	5	10	3	8	1	6
Bob:	12	3	6	9	12	3	6

So Alice and Bob will be together again after six moves.

OR

If Bob does not move and Alice moves $9 + 5 = 14$ points or 2 points each time, they will still be in the same relative position from each other after each turn. If Bob does not move, they will be on the same point when Alice first stops on point 12, where she started. So Alice will have to move 2 steps 6 times to stop at her starting point.

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2016 Q20

20. The least common multiple of a and b is 12, and the least common multiple of b and c is 15. What is the least possible value of the least common multiple of a and c?

(A) 20 **(B)** 30 **(C)** 60 **(D)** 120 **(E)** 180

20. Answer (A):

If $b = 1$, then $a = 12$ and $c = 15$, and the least common multiple of a and c is 60. If $b > 1$, then any prime factor of b must also be a factor of both 12 and 15, and thus the only possible value is $b = 3$. In this case, a must be a multiple of 4 and a divisor of 12, so $a = 4$ or $a = 12$. Similarly, c must be a multiple of 5 and a divisor of 15, so $c = 5$ or $c = 15$. It follows that the least common multiple of a and c must be a multiple of 20. When $a = 4$, $b = 3$, and $c = 5$, the least common multiple of a and c is exactly 20.