2006 Q11

11. How many two-digit numbers have digits whose sum is a perfect square?

(A) 13 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

11. **(C)** The sum of the digits of a two-digit number is at most $9 + 9 = 18$. This means the only possible perfect square sums are $1, 4, 9$ and 16. Each square has the following two-digit possibilities:
$1 : 10$
$4 : 40, 31, 22, 13$
$9 : 90, 81, 72, 63, 54, 45, 36, 27, 18$
$16 : 97, 88, 79$
There are 17 two-digit numbers in all.

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2016 Q11

11. Determine how many two-digit numbers satisfy the following property: When the number is added to the number obtained by reversing its digits, the sum is 132.

(A) 5 **(B)** 7 **(C)** 9 **(D)** 11 **(E)** 12

11. Answer (B):

Let $\underline{a}\,\underline{b}$ be the two digit number. Then $132 = (10a + b) + (10b + a) = 11(a + b)$.

Thus $a + b = 12$. The possible numbers are: 39, 93, 48, 84, 57, 75, and 66. There are seven two-digit numbers that meet this criterion.

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1995 Q12

12. A *lucky* year is one in which at least one date, when written in the form month/day/year, has the following property: *The product of the month times the day equals the last two digits of the year.* For example, 1956 is a lucky year because it has the date 7/8/56 and $7 \times 8 = 56$. Which of the following is NOT a lucky year?

(A) 1990 **(B)** 1991 **(C)** 1992 **(D)** 1993 **(E)** 1994

12. **(E)** The last two digits of 1994 can only be factored as $94 = 2 \times 47$. All the other choices have at least one date that makes them lucky:
(A) 9/10/90 **(B)** 7/13/91 **(C)** 4/23/92 **(D)** 3/31/93

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1991 Q12

12. If $\dfrac{2+3+4}{3} = \dfrac{1990+1991+1992}{N}$, then $N =$

(A) 3 **(B)** 6 **(C)** 1990 **(D)** 1991 **(E)** 1992

12. (D) Any fraction of the form $\frac{(k-1)+k+(k+1)}{k}$ equals 3, since $(k-1)+k+(k+1)=3k$ and $\frac{3k}{k}=3$. The denominator of the fraction must equal the middle term of the numerator. Thus $N = 1991$.

OR

Note that $\frac{2+3+4}{3}=\frac{9}{3}=\frac{3\times 3}{3}=3\times\frac{3}{3}=3\times 1=3$.

Also, $\frac{1990+1991+1992}{N}=\frac{3\times 1991}{N}=3\times\frac{1991}{N}=3\times 1=3$.

Thus N must equal 1991.

2003 Q12

12. When a fair six-sided die is tossed on a table top, the bottom face cannot be seen. What is the probability that the product of the numbers on the five faces that can be seen is divisible by 6?

(A) $\frac{1}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{5}{6}$ **(E)** 1

12. **(E)** If 6 is one of the visible faces, the product will be divisible by 6. If 6 is not visible, the product of the visible faces will be $1\times 2\times 3\times 4\times 5=120$, which is also divisible by 6. Because the product is always divisible by 6, the probability is 1.

1992 Q12

12. The five tires of a car (four road tires and a full-sized spare) were rotated so that each tire was used the same number of miles during the first 30,000 miles the car traveled. For how many miles was each tire used?

(A) 6000 (B) 7500 (C) 24,000 (D) 30,000 (E) 37,500

12. (C) The total number of miles of wear is $30,000 \times 4 = 120,000$. Since this wear is shared equally by each of the 5 tires, each tire traveled $120,000 \div 5 = 24,000$ miles.

OR

Since each of the tires was on for $\frac{4}{5}$ of the driving, it follows that each was used $\frac{4}{5} \times 30,000 = 24,000$ miles.

12. What is the units digit of 13^{2012}?

2012 Q12

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

12. **Answer (A):** To determine the units digit of 13^{2012}, looking for patterns is a good approach to finding the solution. The units digit of each power of 13 depends only on the units digit of the previous power, as follows:
For 13^1, the units digit is 3.
For 13^2, $3 \times 3 = 9$, so the units digit is 9.
For 13^3, $9 \times 3 = 27$, so the units digits is 7.
For 13^4, $7 \times 3 = 21$, so the units digit is 1.
For 13^5, $1 \times 3 = 3$, so the units digit is 3.
The units digits of successive powers of 13 follow the pattern 3, 9, 7, 1, 3, 9, 7, 1, ...
Since $2012 = 4 \times 503$, the units digit of 13^{2012} is 1.

2017 Q12

12. The smallest positive integer greater than 1 that leaves a remainder of 1 when divided by 4, 5, and 6 lies between which of the following pairs of numbers?

(A) 2 and 19 **(B)** 20 and 39 **(C)** 40 and 59

(D) 60 and 79 **(E)** 80 and 124

12. **Answer (D):** The least common multiple of 4, 5, and 6 is 60. Numbers that leave a remainder of 1 when divided by 4, 5, and 6 are 1 more than a whole number multiple of 60. So the smallest positive number greater than 1 that leaves a remainder of 1 when divided by 4, 5, and 6 is 61.

2013 Q13

13. When Clara totaled her scores, she inadvertently reversed the units digit and the tens digit of one score. By which of the following might her incorrect sum have differed from the correct one?

(A) 45 **(B)** 46 **(C)** 47 **(D)** 48 **(E)** 49

13. **Answer (A):** Switching a digit from the units column to the tens column increases the sum by 9 times the value of that digit. For example, switching a 7 from the units column to the tens column increases the sum by $70-7=63=9\cdot 7$. Similarly, switching a digit from the tens column to the units column decreases the sum by 9 times the value of that digit. Therefore reversing two digits changes the sum by an amount that must be a multiple of 9. Among the given choices, only 45 is a possible difference.

(Note: Other multiples of 9 are also possible.)

14. If $200 \leq a \leq 400$ and $600 \leq b \leq 1200$, then the largest value of the quotient $\frac{b}{a}$ is

A) $\frac{3}{2}$ B) 3 C) 6 D) 300 E) 600

1986 Q14

14. (C) The maximum value for the quotient $\frac{b}{a}$ is formed by choosing the largest possible value for b and the smallest possible value for a, or $\frac{1200}{200} = 6$.

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1991 Q14

14. Several students are competing in a series of three races. A student earns 5 points for winning a race, 3 points for finishing second and 1 point for finishing third. There are no ties. What is the smallest number of points that a student must earn in the three races to be guaranteed of earning more points than any other student?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 13 **(E)** 15

14. **(D)** If one student earns $5+5+5=15$ points, no other student can earn more than $3+3+3=9$ points.
If one student earns $5+5+3=13$ points, no other student can earn more than $3+3+5=11$ points.
However, if one student earns $5+3+3=11$ or $5+5+1=11$ points, some other student can earn $3+5+5=13$ or $3+3+5=11$ points.
Thus 13 points is the smallest number of points a student must earn to be guaranteed of earning more points than any other student.

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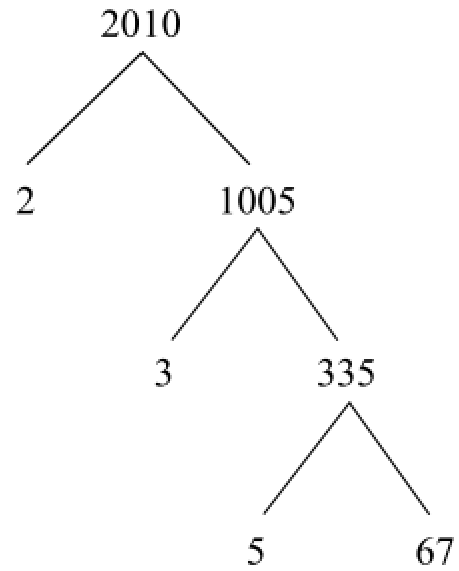
14. What is the sum of the prime factors of 2010 ?

2010 Q14

(A) 67 **(B)** 75 **(C)** 77 **(D)** 201 **(E)** 210

14. **Answer (C):** The prime factors of 2010 are: 2, 3, 5, 67



The sum of the prime factors is $2 + 3 + 5 + 67 = 77$.

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2015 Q14

14. Which of the following integers cannot be written as the sum of four consecutive odd integers?

(A) 16 **(B)** 40 **(C)** 72 **(D)** 100 **(E)** 200

14. **Answer (D):** The sum of 4 consecutive odd integers is always a multiple of 8, $(2n-3)+(2n-1)+(2n+1)+(2n+3)=8n$. Among the given choices, only 100 is not a multiple of 8. The other four numbers can each be written as the sum of four consecutive odd numbers:

$$16 = 1+3+5+7$$
$$40 = 7+9+11+13$$
$$72 = 15+17+19+21$$
$$200 = 47+49+51+53$$

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14. What is the units digit of $19^{19}+99^{99}$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 8 **(E)** 9

2000 Q14

14. **Answer (D):** The units digit of a power of an integer is determined by the units digit of the integer; that is, the tens digit, hundreds digit, etc... of the integer have no effect on the units digit of the result. In this problem, the units digit of 19^{19} is the units digit of 9^{19}. Note that $9^1=9$ ends in 9, $9^2=81$ ends in 1, $9^3=729$ ends in 9, and, in general, the units digit of odd powers of 9 is 9, whereas the units digit of even powers of 9 is 1. Since both exponents are odd, the sum of their units digits is $9+9=18$, the units digit of which is 8.

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2007 Q15

15. Let a, b and c be numbers with $0<a<b<c$. Which of the following is impossible?

(A) $a+c<b$ **(B)** $a\cdot b<c$ **(C)** $a+b<c$ **(D)** $a\cdot c<b$ **(E)** $\frac{b}{c}=a$

15. **(A)** Because $b < c$ and $0 < a$, adding corresponding sides of the inequalities gives $b < a + c$, so (A) is impossible. To see that the other choices are possible, consider the following choices for a, b, and c:

(B) and (C): $a = 1$, $b = 2$, and $c = 4$;

(D): $a = \frac{1}{3}$, $b = 1$, and $c = 2$;

(E): $a = \frac{1}{2}$, $b = 1$, and $c = 2$.

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2012 Q15

15. The smallest number greater than 2 that leaves a remainder of 2 when divided by 3, 4, 5, or 6 lies between what numbers?

(A) 40 and 50 **(B)** 51 and 55 **(C)** 56 and 60 **(D)** 61 and 65

(E) 66 and 99

15. **Answer (D):** Two less than the number must be divisible by 3, 4, 5, and 6. The least common multiple of these numbers is 60, so 62 is the smallest number greater than 2 to leave a remainder of 2 when divided by 3, 4, 5, and 6. Therefore the number lies between 61 and 65.

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2011 Q15

15. How many digits are in the product $4^5 \cdot 5^{10}$?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 15

15. **Answer (D):** The product $4^5 \cdot 5^{10} = 2^{10} \cdot 5^{10} = 10^{10}$ is a number with a 1 followed by 10 zeros for a total of 11 digits.

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2013 Q15

15. If $3^p + 3^4 = 90$, $2^r + 44 = 76$, and $5^3 + 6^s = 1421$, what is the product of p, r, and s?

(A) 27 **(B)** 40 **(C)** 50 **(D)** 70 **(E)** 90

15. **Answer (B):** From the first equation, $3^p+81 = 90$, so $3^p = 9$, and $p = 2$. From the second equation, $2^r = 32$, so $r = 5$. From the third equation, $6^s+125 = 1421$, so $6^s = 1296$, and $s = 4$. The product of p, r, and s is $2 \cdot 5 \cdot 4 = 40$.

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2016 Q15

15. What is the largest power of 2 that is a divisor of $13^4 - 11^4$?

(A) 8 **(B)** 16 **(C)** 32 **(D)** 64 **(E)** 128

15. Answer (C):

Factor, using a difference of two squares:

$$\begin{aligned} 13^4 - 11^4 &= \left(13^2 + 11^2\right)(13+11)(13-11) \\ &= 290 \cdot 24 \cdot 2 \\ &= 2 \cdot 145 \cdot 8 \cdot 3 \cdot 2 \\ &= 32 \cdot 435 \end{aligned}$$

So the largest power of 2 that is a divisor of $13^4 - 11^4$ is 32.