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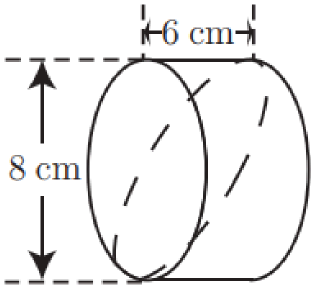
2008 Q21

21. Jerry cuts a wedge from a 6-cm cylinder of bologna as shown by the dashed curve. Which answer choice is closest to the volume of his wedge in cubic centimeters?



(A) 48 **(B)** 75 **(C)** 151 **(D)** 192 **(E)** 603

21. **Answer (C):** Using the formula for the volume of a cylinder, the bologna has volume $\pi r^2 h = \pi \times 4^2 \times 6 = 96\pi$. The cut divides the bologna in half. The half-cylinder will have volume $\frac{96\pi}{2} = 48\pi \approx 151\ \text{cm}^3$.
Note: The value of π is slightly greater than 3, so to estimate the volume multiply $48(3) = 144\ \text{cm}^3$. The product is slightly less than and closer to answer C than any other answer.

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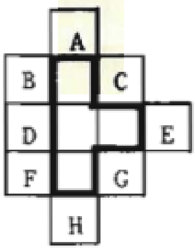
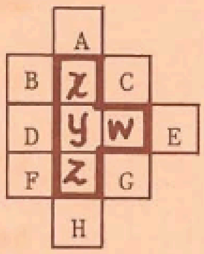
1986 Q21

21. Suppose one of the eight lettered identical squares is included with the four squares in the T-shaped figure outlined. How many of the resulting figures can be folded into a topless cubical box?



A) 2 B) 3 C) 4 D) 5 E) 6

21. (E) Label the four squares in the T-shaped figure X,Y,Z,W as shown. First think of W as the base of the cube. Then X,Y,Z will be three of the "sides" and the fourth side could be A, E, or H. Now think of Y as the base. Then X,W,Z are three sides and B, D, or F could be the fourth side. C and G are not possible because four sides of a cube cannot come together at a point.



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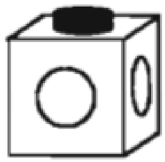
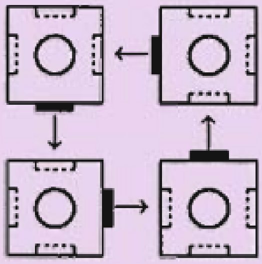
1995 Q21

21. A plastic snap-together cube has a protruding snap on one side and receptacle holes on the other five sides as shown. What is the smallest number of these cubes that can be snapped together so that only receptacle holes are showing?



(A) 3 (B) 4 (C) 5 (D) 6 (E) 8

21. **(B)** Using one, two or three cubes always leaves one protuding snap showing. The smallest number of cubes is four, arranged as shown (viewed from above).



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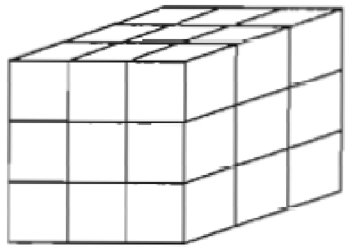
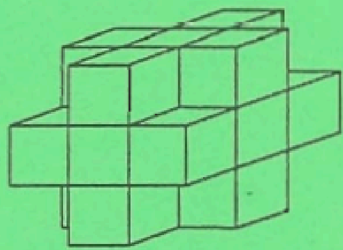
1997 Q21

21. Each corner cube is removed from this 3 cm x 3 cm x 3 cm cube. The surface area of the remaining figure is



(A) 19 sq.cm **(B)** 24 sq.cm **(C)** 30 sq.cm **(D)** 54 sq.cm **(E)** 72 sq.cm

21. **(D)** In terms of the original cube, three square centimeters are lost from each corner, but three new squares are added as the sides of the cavity in that corner. The total area remains at 54 square centimenters.



OR

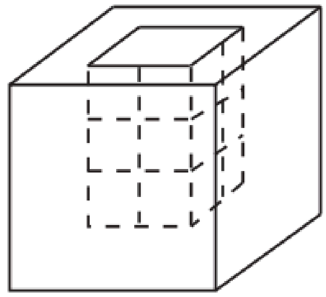
Each face had an area of 9 square centimeters originally, so the total area was 6 (9) = 54 sq. cm. Four squares are lost from each face, leaving 54 - 4 (6) = 30 sq. cm. But the cavity in each of the eight corners has an area of 3 sq. cm, so 8 (3) = 24 sq. cm must be added. Finally, 30 + 24 = 54 sq. cm.

1998 Q21

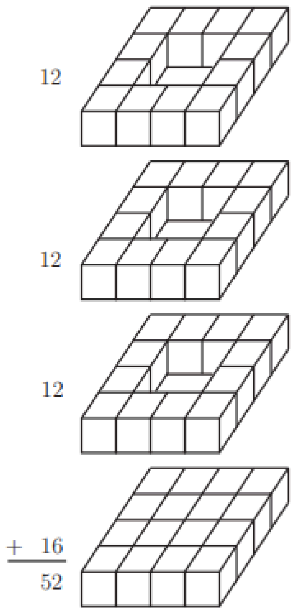
21. A $4 \times 4 \times 4$ cubical box contains 64 identical small cubes that exactly fill the box. How many of these small cubes touch a side or the bottom of the box?

(A) 48 **(B)** 52 **(C)** 60 **(D)** 64 **(E)** 80

21. **Answer (B):** The $2 \times 2 \times 3$ core contains all of the small cubes that do not touch a side or the bottom. These 12 cubes are subtracted from 64 to leave 52.



OR



2012 Q21

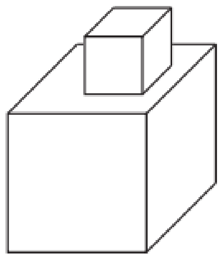
21. Marla has a large white cube that has an edge of 10 feet. She also has enough green paint to cover 300 square feet. Marla uses all the paint to create a white square centered on each face, surrounded by a green border. What is the area of one of the white squares, in square feet?

(A) $5\sqrt{2}$ **(B)** 10 **(C)** $10\sqrt{2}$ **(D)** 50 **(E)** $50\sqrt{2}$

21. **Answer (D):** The surface area of the cube is $6 \times 10^2 = 600$ square feet. The green paint covers 300 square feet, so the total area of the white squares is $600 - 300 = 300$ square feet. There are 6 white squares, so each has area $\frac{300}{6} = 50$ square feet.

2000 Q22

22. A cube has edge length 2. Suppose that we glue a cube of edge length 1 on top of the big cube so that one of its faces rests entirely on the top face of the larger cube. The percent increase in the surface area (sides, top, and bottom) from the original cube to the new solid formed is closest to:



(A) 10 **(B)** 15 **(C)** 17 **(D)** 21 **(E)** 25

22. **Answer (C):** The area of each face of the larger cube is $2^2 = 4$. There are six faces of the cube, so its surface area is $6(4) = 24$. When we add the smaller cube, we decrease the original surface area by 1, but we add $5(1^2) = 5$ units of area (1 unit for each of the five unglued faces of the smaller cube), This is a net increase of 4 from the original surface area, and 4 is $\frac{4}{24} = \frac{1}{6} \approx 16.7\%$ of 24. The closest value given is 17.

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1997 Q22

22. A two-inch cube (2x2x2) of silver weighs 3 pounds and is worth \$200. How much is a three-inch cube of silver worth?

(A) \$300 **(B)** \$375 **(C)** \$450 **(D)** \$560 **(E)** \$675

22. **(E)** The volume of a two-inch cube is $2^3 = 8$ cu. inches, while that of a three-inch cube is 27 cu. inches. Therefore, the weight and value of the larger cube is $\frac{27}{8}$ times that of the smaller. \$200 ($\frac{27}{8}$) = \$675.
Note: The actual weight of the cubes is not needed to solve the problem.

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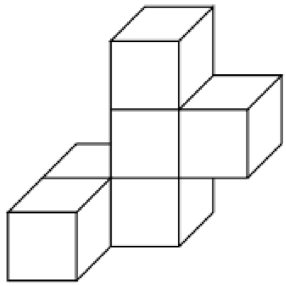
2002 Q22

22. Six cubes, each an inch on an edge, are fastened together, as shown. Find the total surface area in square inches. Include the top, bottom and sides.

(A) 18 **(B)** 24 **(C)** 26 **(D)** 30 **(E)** 36



22. **(C)** When viewed from the top and bottom, there are 4 faces exposed; from the left and right sides, there are 4 faces exposed and from the front and back, there are 5 faces exposed. The total is $4+4+4+4+5+5 = 26$ exposed faces.

OR

Before the cubes were glued together, there were $6 \times 6 = 36$ faces exposed. Five pairs of faces were glued together, so $5 \times 2 = 10$ faces were no longer exposed. This leaves $36 - 10 = 26$ exposed faces.

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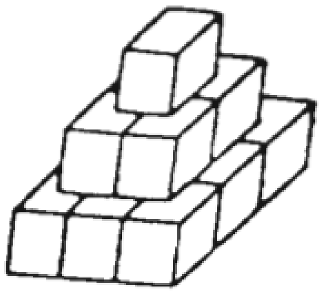
1989 Q23

23. An artist has 14 cubes, each with an edge of 1 meter. She stands them on the ground to from a sculpture as shown. She then paints the exposed surface of the sculpture. How many square meters does she paint?

A) 21 B) 24 C) 33 D) 37 E) 42



23. C There are 6 faces exposed on each of the four sides for a total of $6 \times 4 = 24$ square meters. From above, there are a total of 9 square meters exposed since the cubes in the second and third layers essentially cover the unexposed portion of the bottom layer. So there are a total of $24 + 9 = 33$ square meters to paint.

OR

The bottom layer has 12 complete faces, four "$\frac{1}{2}$ faces", and four "$\frac{3}{4}$ faces".

The second layer has 8 complete faces and four "$\frac{3}{4}$ faces". The top layer has 5 complete faces. The total, then, is

$12 + 4(\frac{1}{2}) + 4(\frac{3}{4}) + 8 + 4(\frac{3}{4}) + 5 = 33$ square meters.

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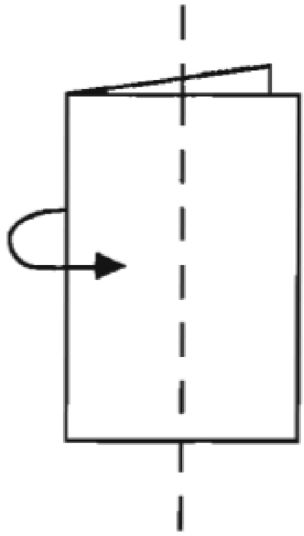
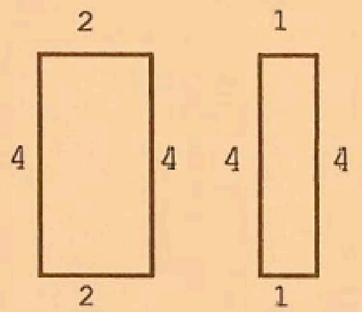
1989 Q24

24. Suppose a square piece of paper is folded in half vertically. The folded paper is then cut in half along the dashed line. Three rectangles are formed–a large one and two small ones. What is the ratio of the perimeter of one of the small rectangles to the perimeter of the large rectangle?

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{4}{5}$ E) $\frac{5}{6}$



24. E



If we let the side of the original square be 4 units, then the large rectangle and one of the small rectangles formed have the dimensions shown. Therefore the ratio of the perimeters is

$$\frac{1+4+4+1}{2+4+4+2} = \frac{10}{12} = \frac{5}{6}.$$

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1991 Q24

24. A cube of edge 3 cm is cut into N smaller cubes, not all the same size. If the edge of each of the smaller cubes is a whole number of centimeters, then $N =$

(A) 4 **(B)** 8 **(C)** 12 **(D)** 16 **(E)** 20

24. **(E)** Since the edge of each smaller cube must be a whole number, the smaller cubes must be $1 \times 1 \times 1$ or $2 \times 2 \times 2$ cubes. There can be only one smaller cube of edge 2, so the rest of the smaller cubes have edge 1. Since the volume of the original cube was $3 \times 3 \times 3 = 27$ cubic cm, and the volume of the cube of edge 2 is $2 \times 2 \times 2 = 8$ cubic cm, then there must be $27 - 8 = 19$ cubes of edge 1 (volume = 1 cubic cm). There are a total of 20 cubes so $N = 20$.

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1994 Q24

24. A 2 by 2 square is divided into four 1 by 1 squares. Each of the small squares is to be painted either green or red. In how many different ways can the painting be accomplished so that no green square shares its top or right side with any red square? There may be as few as zero or as many as four small green squares.

(A) 4 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 16

24. (B) If there is any square painted green, then all the squares above or to the right of it must also be green. Therefore, the possible patterns of colors are:

$$\begin{bmatrix} R & R \\ R & R \end{bmatrix} \quad \begin{bmatrix} R & G \\ R & R \end{bmatrix} \quad \begin{bmatrix} G & G \\ R & R \end{bmatrix}$$
$$\begin{bmatrix} R & G \\ R & G \end{bmatrix} \quad \begin{bmatrix} G & G \\ R & G \end{bmatrix} \quad \begin{bmatrix} G & G \\ G & G \end{bmatrix}$$

Thus, there are 6 different ways to paint the squares according to the requirements of the problem.

OR

All the green squares on any row must be to the right. The number of green squares in any row must be at least as large as the number of green squares in any lower row. Therefore, the number of ways to paint n squares green is the number of sums $a + b = n$ with $2 \geq a \geq b \geq 0$, and n can be any number from 0 through 4. There are 6 such sums:

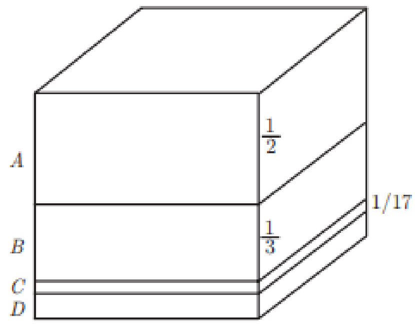
$$0+0=0 \qquad 1+0=1 \qquad 2+0=2$$
$$1+1=2 \qquad 2+1=3 \qquad 2+2=4$$

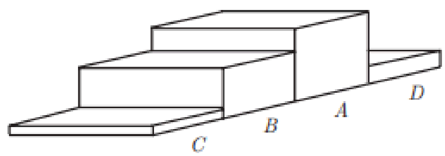
Therefore, there are 6 ways to paint the 2 by 2 square according to the requirements of the problem.

Query. What if the original square were 3 by 3? 4 by 4?

2009 Q25

25. A one-cubic-foot cube is cut into four pieces by three cuts parallel to the top face of the cube. The first cut is $\frac{1}{2}$ foot from the top face. The second cut is $\frac{1}{3}$ foot below the first cut, and the third cut is $\frac{1}{17}$ foot below the second cut. From the top to the bottom the pieces are labeled A, B, C and D. The pieces are then glued together end to end in the order C, B, A, D to make a long solid as shown below. What is the total surface area of this solid in square feet?





(A) 6 **(B)** 7 **(C)** $\frac{419}{51}$ **(D)** $\frac{158}{17}$ **(E)** 11

25. **Answer (E):** Looking from either end, the visible area totals $\frac{1}{2}$ square foot because piece A measures $\frac{1}{2} \times 1 = \frac{1}{2}$ ft^2, and the other pieces decrease in height from that piece. The two side views each show four blocks that can stack to a unit cube. So the area as seen from each side is 1 ft^2. Finally, the top and bottom views each show four unit squares. So the top and bottom view each contribute 4 ft^2 to the area. Summing, the total surface area is

$$\frac{1}{2} + \frac{1}{2} + 1 + 1 + 4 + 4 = 11 \text{ square feet.}$$

CHALLENGE: Suppose the cuts are $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$. Does this change the solution?