2010 Q21

21. Hui is an avid reader. She bought a copy of the best-seller *Math is Beautiful*. On the first day, Hui read $\frac{1}{5}$ of the pages plus 12 more, and on the second day she read $\frac{1}{4}$ of the remaining pages, plus 15 pages. On the third day, she read $\frac{1}{3}$ of the remaining pages, plus 18 pages. She then realized that there were only 62 pages left to read, which she read the next day. How many pages are in this book?

(A) 120 **(B)** 180 **(C)** 240 **(D)** 300 **(E)** 360

21. **Answer (C):** Reason backward as follows. On the third day, Hui reads $\frac{1}{3}$ of the remaining pages plus 18 more, leaving her with 62 pages left to read. This means that $62+18=80$ is $\frac{2}{3}$ of the number of pages remaining at the end of the second day. She had $\frac{3}{2} \times 80 = 120$ pages left to read at the end of the second day. On the second day she read $\frac{1}{4}$ of the remaining pages plus 15 more, so $120 + 15 = 135$ is $\frac{3}{4}$ of the number of pages remaining at the end of the first day. She had $\frac{4}{3} \times 135 = 180$ pages left to read at the end of the first day. On the first day she read $\frac{1}{5}$ of the pages plus 12 more. So $180 + 12 = 192$ is $\frac{4}{5}$ of the number of pages in the book. The total number of pages is $\frac{5}{4} \times 192 = 240$.

OR

Setup a chart working backwards through the days, to show the number of pages that Hui has left to read each day.

Day	Extra Pages Read	Fraction Read	Fraction Left	Calculation	Pages to be Read
4	0				62
3	18	1/3	2/3	$(62+18)/(2/3)$	120
2	15	1/4	3/4	$(120+15)/(3/4)$	180
1	12	1/5	4/5	$(180+12)/(4/5)$	240

1985 Q23

23. King Middle School has 1200 students. Each student takes 5 classes a day. Each teacher teaches 4 classes. Each class has 30 students and 1 teacher. How many teachers are there at King Middle School?

A) 30 B) 32 C) 40 D) 45 E) 50

23. (E) There are $1200 \times 5 = 6000$ times a day a student attends a class. Thus there are $\frac{6000}{30} = 200$ times a day a teacher teaches a class, so there must be $\frac{200}{4} = 50$ teachers.

OR

If each student took 4 classes and each teacher taught 4 classes, then $\frac{1200}{30} = 40$ teachers would be required. But each student takes 5 classes, so $\frac{5}{4} \times 40 = 50$ teachers are needed.

2006 Q23

23. A box contains gold coins. If the coins are equally divided among six people, four coins are left over. If the coins are equally divided among five people, three coins are left over. If the box holds the smallest number of coins that meets these two conditions, how many coins are left when equally divided among seven people?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 5

23. **(A)** The counting numbers that leave a remainder of 4 when divided by 6 are $4, 10, 16, 22, 28, 34, \ldots$. The counting numbers that leave a remainder of 3 when divided by 5 are $3, 8, 13, 18, 23, 28, 33, \ldots$. So 28 is the smallest possible number of coins that meets both conditions. Because $4 \times 7 = 28$, there are no coins left when they are divided among seven people.

OR

If there were two more coins in the box, the number of coins would be divisible by both 6 and 5. The smallest number that is divisible by 6 and 5 is 30, so the smallest possible number of coins in the box is 28.

4 / 5

1992 Q25

25. One half of the water is poured out of a full container. Then one third of the remainder is poured out. Continue the process: one fourth of the remainder for the third pouring, one fifth of the remainder for the fourth pouring, etc. After how many pourings does exactly one tenth of the original water remain?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

25. **(D)** After the first pouring, $\frac{1}{2}$ remains. After the second pouring $\frac{1}{2} \times \frac{2}{3}$ remains. After the third pouring $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$ remains. How many pourings until $\frac{1}{10}$ remains?

$$\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{6}} \times \frac{\cancel{6}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{8}} \times \frac{\cancel{8}}{\cancel{9}} \times \frac{\cancel{9}}{10} = \frac{1}{10}$$

indicates 9 pourings.

OR

Make a table for the information:

Pouring	Amount Poured	Amount Remaining
1	$\frac{1}{2}$	$1 - \frac{1}{2} = \frac{1}{2}$
2	$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$	$\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$
3	$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$	$\frac{1}{3} - \frac{1}{12} = \frac{1}{4}$
4	$\frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$	$\frac{1}{4} - \frac{1}{20} = \frac{1}{5}$
⋮	⋮	⋮
n	$\frac{1}{n+1} \times \frac{1}{n} = \frac{1}{n(n+1)}$	$\frac{1}{n} - \frac{1}{n(n+1)} = \frac{1}{n+1}$
⋮	⋮	⋮
9	$\frac{1}{10} \times \frac{1}{9} = \frac{1}{90}$	$\frac{1}{9} - \frac{1}{90} = \frac{1}{10}$

Thus 1/10 remains after the 9^{th} pouring.

2002 Q25

25. Loki, Moe, Nick and Ott are good friends. Ott had no money, but the others did. Moe gave Ott one-fifth of his money, Loki gave Ott one-fourth of his money and Nick gave Ott one-third of his money. Each gave Ott the same amount of money. What fractional part of the group's money does Ott now have?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{1}{2}$

25. **(B)** Only the fraction of each friend's money is important, so we can assume any convenient amount is given to Ott. Suppose that each friend gave Ott \$1. If this is so, then Moe had \$5 orignally and now has \$4, Loki had \$ 4 and now has \$3, and Nick had \$3, and now has \$2. The four friends now have $\$4 + \$3 + \$2 + \$3 = \$12$, so Ott has $\frac{3}{12} = \frac{1}{4}$ of the group's money. This same reasoning applies to any amount of money.