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1989 Q16

16. In how many ways can 47 be written as the sum of two primes?

A) 0 B) 1 C) 2 D) 3 E) more than 3

16. A It is not possible to write 47 as the sum of two primes. When 47 is written as the sum of two whole numbers, one must be even and the other odd. Since 2 is the only even prime, there is only one case to consider. In that case, the other summand must be 45 which is not a prime since it has a factor of 5.

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1986 Q17

17. Let o be an odd whole number and let n be any whole number. Which of the following statements about the whole number $(o^2 + no)$ is always true?

A) it is always odd B) it is always even

C) it is even only if n is even D) it is odd only if n is odd

E) it is odd only if n is even

17. (E) The number o^2 is always odd. Now (no) is odd if n is odd and even if n is even. Thus the sum $(o^2 + no)$ is even if n is odd, since the sum of two odd numbers is even; the sum is odd if n is even, since the sum of an odd number and an even number is odd.

2009 Q17

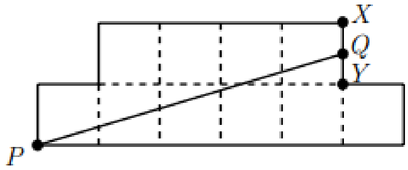
17. The positive integers x and y are the two smallest positive integers for which the product of 360 and x is a square and the product of 360 and y is a cube. What is the sum of x and y?

(A) 80 **(B)** 85 **(C)** 115 **(D)** 165 **(E)** 610

17. **Answer (B):** Factor 360 into $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$. First increase the number of each factor as little as possible to form a square: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 = (2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5)(2 \cdot 5) = (360)(10)$, so x is 10. Then increase the number of each factor as little as possible to form a cube: $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = (2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5)(3 \cdot 5 \cdot 5) = (360)(75)$, so y is 75. The sum of x and y is $10 + 75 = 85$.

2010 Q17

17. The diagram shows an octagon consisting of 10 unit squares. The portion below $\overline{PQ}$ is a unit square and a triangle with base 5. If $\overline{PQ}$ bisects the area of the octagon, what is the ratio $\frac{XQ}{QY}$?



(A) $\frac{2}{5}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

17. **Answer (D):** The area below $\overline{PQ}$ is

$$1+\frac{1}{2}\cdot 5\cdot(1+QY)=5$$
$$\frac{5}{2}\cdot(1+QY)=4$$
$$1+QY=\frac{8}{5}$$

$$QY=\frac{3}{5}$$

Then $XQ=1-QY=1-\frac{3}{5}=\frac{2}{5}$, so $\frac{XQ}{QY}=\frac{2/5}{3/5}=\frac{2}{3}$.

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2011 Q17

17. Let w, x, y, and z be whole numbers. If $2^w\cdot 3^x\cdot 5^y\cdot 7^z=588$, then what does $2w+3x+5y+7z$ equal?

(A) 21 **(B)** 25 **(C)** 27 **(D)** 35 **(E)** 56

17. **Answer (A):** Factor 588 into $2^2 \cdot 3^1 \cdot 5^0 \cdot 7^2$. Thus $w = 2, x = 1, y = 0$, and $z = 2$, and $2w + 3x + 5y + 7z = 21$.

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2005 Q18

18. How many three-digit numbers are divisible by 13?

(A) 7 **(B)** 67 **(C)** 69 **(D)** 76 **(E)** 77

18. **(C)** The smallest three-digit number divisible by 13 is $13 \times 8 = 104$, so there are seven two-digit multiples of 13. The greatest three-digit number divisible by 13 is $13 \times 76 = 988$. Therefore, there are $76 - 7 = 69$ three-digit numbers divisible by 13.

OR

Because the integer part of $\frac{999}{13}$ is 76, there are 76 multiples of 13 less than or equal to 999. Because the integer part of $\frac{99}{13}$ is 7, there are 7 multiples of 13 less than or equal to 99. That means there are $76 - 7 = 69$ multiples of 13 between 100 and 999.

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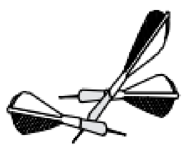
2004 Q18

18. Five friends compete in a dart-throwing contest. Each one has two darts to throw at the same circular target, and each individual's score is the sum of the scores in the target regions that are hit. The scores for the target regions are the whole numbers 1 through 10. Each throw hits the target in a region with a different value. The scores are: Alice 16 points, Ben 4 points, Cindy 7 points, Dave 11 points, and Ellen 17 points. Who hits the region worth 6 points?



(A) Alice **(B)** Ben **(C)** Cindy **(D)** Dave **(E)** Ellen

18. **(A)** Ben must hit 1 and 3. This means Cindy must hit 5 and 2, because she scores 7 using two different numbers, neither of which is 1 or 3. By similar reasoning, Alice hits 10 and 6, Dave hits 7 and 4, and Ellen hits 9 and 8. Alice hits the 6.

OR

Ellen's score can be obtained by either $10 + 7$ or $9 + 8$. In the first case, it is impossible for Alice to score 16. So Ellen's 17 is obtained by scoring 9 and 8, and Alice's total of 16 is the result of her hitting 10 and 6. The others scored $11 = 7 + 4$, $7 = 5 + 2$ and $4 = 3 + 1$.

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2007 Q18

18. The product of the two 99-digit numbers

$$303{,}030{,}303,\ldots,030{,}303 \quad \text{and} \quad 505{,}050{,}505,\ldots,050{,}505$$

has thousands digit A and units digit B. What is the sum of A and B?

(A) 3 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 10

18. **(D)** To find A and B, it is sufficient to consider only $303 \cdot 505$, because 0 is in the thousands place in both factors.

$$\begin{array}{r} \cdots 303 \\ \times \quad \cdots 505 \\ \hline \cdots 1515 \\ \cdots 1500 \\ \hline \cdots 3015 \end{array}$$

So $A = 3$ and $B = 5$, and the sum is $A + B = 3 + 5 = 8$.

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2012 Q18

18. What is the smallest positive integer that is neither prime nor square and that has no prime factor less than 50?

(A) 3127 **(B)** 3133 **(C)** 3137 **(D)** 3139 **(E)** 3149

18. **Answer (A):** Since the integer is neither prime nor square, it is divisible either by two distinct primes or by the cube of a prime. The smallest prime numbers not less than 50 are 53 and 59. Since $53 \times 59 = 3127 < 53^3$, the smallest number satisfying this description is 3127.

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2004 Q19

19. A whole number larger than 2 leaves a remainder of 2 when divided by each of the numbers 3, 4, 5 and 6. The smallest such number lies between which two numbers?

(A) 40 and 49 **(B)** 60 and 79 **(C)** 100 and 129 **(D)** 210 and 249 **(E)** 320 and 369

19. **(B)** The numbers that leave a remainder of 2 when divided by 4 and 5 are 22, 42, 62 and so on. Checking these numbers for a remainder of 2 when divided by both 3 and 6 yields 62 as the smallest.

OR

The smallest whole number that is evenly divided by each of 3, 4, 5 and 6 is $\text{LCM}\{3,4,5,6\} = 2^2 \times 3 \times 5 = 60$. So the smallest whole number greater than 2 that leaves a remainder of 2 when divided by each of 3, 4, 5 and 6 is 62.