1993 Q16

16. $\dfrac{1}{1+\dfrac{1}{2+\dfrac{1}{3}}} =$

(A) $\frac{1}{6}$ (B) $\frac{3}{10}$ (C) $\frac{7}{10}$ (D) $\frac{5}{6}$ (E) $\frac{10}{3}$

16. (C) Using the common denominators and simplifying yield

$$\frac{1}{1+\dfrac{1}{2+\dfrac{1}{3}}} = \frac{1}{1+\dfrac{1}{\dfrac{7}{3}}} = \frac{1}{1+\dfrac{3}{7}} = \frac{1}{\dfrac{10}{7}} = \frac{7}{10}.$$

OR

Clearing fractions and simplifying yield

$$\frac{1}{1+\dfrac{1\times 3}{\left(2+\dfrac{1}{3}\right)\times 3}} = \frac{1}{1+\dfrac{3}{6+1}} = \frac{1\times 7}{\left(1+\dfrac{3}{7}\right)\times 7} = \frac{7}{7+3} = \frac{7}{10}.$$

OR

Estimating, the result is between

$$\frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3} \quad \text{and} \quad \frac{1}{1+\frac{1}{3}} = \frac{1}{\frac{4}{3}} = \frac{3}{4}.$$

The only choice in this interval is $\frac{7}{10}$.

2004 Q16

16. Two 600 ml pitchers contain orange juice. One pitcher is $\frac{1}{3}$ full and the other pitcher is $\frac{2}{5}$ full. Water is added to fill each pitcher completely, then both pitchers are poured into one large container. What fraction of the mixture in the large container is orange juice?

(A) $\frac{1}{8}$ **(B)** $\frac{3}{16}$ **(C)** $\frac{11}{30}$ **(D)** $\frac{11}{19}$ **(E)** $\frac{11}{15}$

16. **(C)** Because the first pitcher was $\frac{1}{3}$ full of orange juice, after filling with water it contains 200 ml of juice and 400 ml of water. Because the second pitcher was $\frac{2}{5}$ full of orange juice, after filling it contains 240 ml of orange juice and 360 ml of water. In all, the amount of orange juice is 440 ml out of a total of 1200 ml or $\frac{440}{1200} = \frac{11}{30}$ of the mixture.

2015 Q16

16. In a middle-school mentoring program, a number of the sixth graders are paired with a ninth-grade student as a buddy. No ninth grader is assigned more than one sixth-grade buddy. If $\frac{1}{3}$ of all the ninth graders are paired with $\frac{2}{5}$ of all the sixth graders, what fraction of the total number of sixth and ninth graders have a buddy?

(A) $\frac{2}{15}$ **(B)** $\frac{4}{11}$ **(C)** $\frac{11}{30}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{11}{15}$

16. **Answer (B):** 2 out of every 5 sixth graders are paired with a ninth grade buddy, and 2 out of every 6 ninth graders are paired with a sixth grade buddy. (The ratios are now expressed so that the number of sixth graders matches the number of ninth graders.) So 4 out of every 11 students are in the mentoring program. The fraction is $\frac{4}{11}$.

OR

Suppose that n sixth graders are paired with n ninth graders. Then the total number of sixth graders is $\frac{5}{2}n$, the total number of ninth graders is $3n$, and the total number of sixth and ninth graders is $\frac{5}{2}n+3n=\frac{11}{2}n$. There are $2n$ students in the mentoring program, which is $\frac{2n}{\frac{11}{2}n}=\frac{4}{11}$ of the total number of students.

1989 Q18

18. Many calculators have a reciprocal key $\boxed{\frac{1}{x}}$ that replaces the current number displayed with its reciprocal. For example, if the display is $\boxed{4}$ and the $\boxed{\frac{1}{x}}$ key is depressed, then the display becomes $\boxed{.25}$. If $\boxed{32}$ is currently displayed, what is the fewest number of times you must depress the $\boxed{\frac{1}{x}}$ key so the display again reads $\boxed{32}$?

A) 1 B) 2 C) 3 D) 4 E) 5

18. B Depressing the key one time yields the reciprocal of 32 or $\frac{1}{32}$. Thus depressing the key a second time yields the reciprocal of $\frac{1}{32}$ or $\frac{1}{\frac{1}{32}} = 32$.

1987 Q18

18. Half the people in a room left. One third of those remaining started to dance. There were then 12 people who were not dancing. The original number of people in the room was

A) 24 B) 30 C) 36 D) 42 E) 72

18. C The 12 people not dancing are $\frac{2}{3}$ of the people remaining, so 18 people remained. Thus there were 2(18) = 36 people in the room originally.

1986 Q20

20. The value of the expression $\frac{(304)^5}{(29.7)(399)^4}$ is closest to

A) .003 B) .03 C) .3 D) 3 E) 30

20. (D) Estimate each of the quantities to obtain the approximation:

$$\frac{(300)^5}{30(400)^4} = \frac{300}{30}\left(\frac{300}{400}\right)^4 = 10\left(\frac{3}{4}\right)^4 = 10\left(\frac{81}{256}\right)$$

$$\approx 10\left(\frac{1}{3}\right) \approx 3.$$

2004 Q20

20. Two-thirds of the people in a room are seated in three-fourths of the chairs. The rest of the people are standing. If there are 6 empty chairs, how many people are in the room?

(A) 12 **(B)** 18 **(C)** 24 **(D)** 27 **(E)** 36

20. **(D)** Because the 6 empty chairs are $\frac{1}{4}$ of the chairs in the room, there are $6 \times 4 = 24$ chairs in all. The number of seated people is $\left(\frac{3}{4}\right)24 = 18$, and this is $\frac{2}{3}$ of the people present. It follows that

$$\frac{18}{\text{people present}} = \frac{2}{3}.$$

So there are 27 people in the room.

2008 Q20

20. The students in Mr. Neatkin's class took a penmanship test. Two-thirds of the boys and $\frac{3}{4}$ of the girls passed the test, and an equal number of boys and girls passed the test. What is the minimum possible number of students in the class?

(A) 12 **(B)** 17 **(C)** 24 **(D)** 27 **(E)** 36

20. **Answer (B):** Because $\frac{2}{3}$ of the boys passed, the number of boys in the class is a multiple of 3. Because $\frac{3}{4}$ of the girls passed, the number of girls in the class is a multiple of 4. Set up a chart and compare the number of boys who passed with the number of girls who passed to find when they are equal.

Total boys	Boys passed
3	2
6	4
9	6

Total girls	Girls passed
4	3
8	6

The first time the number of boys who passed equals the number of girls who passed is when they are both 6. The minimum possible number of students is $9 + 8 = 17$.

OR

Because $\frac{2}{3}$ of the boys passed, the number of boys who passed must be a multiple of 2. Because $\frac{3}{4}$ of the girls passed, the number of girls who passed must be a multiple of 3. Because the same number of boys and girls passed, the smallest possible number is 6, the least common multiple of 2 and 3. If 6 of 9 boys and 6 of 8 girls passed, there are 17 students in the class, and that is the minimum number possible.

OR

Let G = the number of girls and B = the number of boys. Then $\frac{2}{3}B = \frac{3}{4}G$, so $8B = 9G$. Because 8 and 9 are relatively prime, the minimum number of boys and girls is 9 boys and 8 girls, for a total of $9 + 8 = 17$ students.

2012 Q20

20. What is the correct ordering of the three numbers $\frac{5}{19}$, $\frac{7}{21}$, and $\frac{9}{23}$, in increasing order?

(A) $\frac{9}{23} < \frac{7}{21} < \frac{5}{19}$ **(B)** $\frac{5}{19} < \frac{7}{21} < \frac{9}{23}$ **(C)** $\frac{9}{23} < \frac{5}{19} < \frac{7}{21}$

(D) $\frac{5}{19} < \frac{9}{23} < \frac{7}{21}$ **(E)** $\frac{7}{21} < \frac{5}{19} < \frac{9}{23}$

20. **Answer (B):** Using a common denominator, $\frac{5}{19} = \frac{105}{399}$ and $\frac{7}{21} = \frac{133}{399}$, so $\frac{5}{19} < \frac{7}{21}$.
Also $\frac{7}{21} = \frac{161}{483}$ and $\frac{9}{23} = \frac{189}{483}$, so $\frac{7}{21} < \frac{9}{23}$.

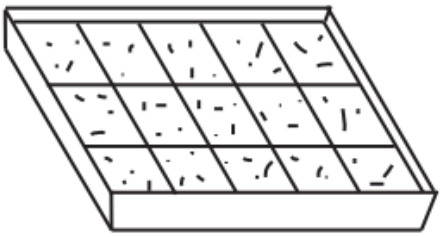
OR

Comparing each fraction, $\frac{7}{21} = \frac{1}{3}$, $\frac{5}{19} < \frac{5}{15} = \frac{1}{3}$, and $\frac{9}{23} > \frac{9}{27} = \frac{1}{3}$, so the correct increasing order is $\frac{5}{19} < \frac{7}{21} < \frac{9}{23}$.

1999 Q17

Problems 17, 18, and 19 refer to the following:

Cookies For a Crowd



At Central Middle School the 108 students who take the AMC $\rightarrow$ 8 meet in the evening to talk about problems and eat an average of two cookies apiece. Walter and Gretel are baking Bonnie's Best Bar Cookies this year. Their recipe, which makes a pan of 15 cookies, list these items:
$1\frac{1}{2}$ cups of flour, 2 eggs, 3 tablespoons butter, $\frac{3}{4}$ cups sugar, and 1 package of chocolate drops. They will make only full recipes, not partial recipe.

17. Walter can buy eggs by the half-dozen. How many half-dozens should be buy to make enough cookies? (Some eggs and some cookies may be left over.)

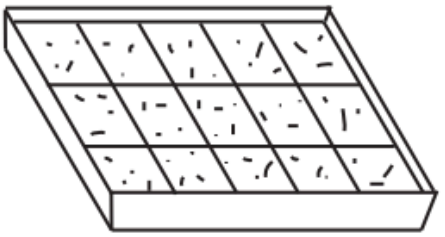
(A) 1 **(B)** 2 **(C)** 5 **(D)** 7 **(E)** 15

17. **Answer (C):** One recipe makes 15 cookies, so $216 \div 15 = 14.4$ recipes are needed, but this must be rounded up to 15 recipes to make enough cookies. Each recipe requires 2 eggs. So 30 eggs are needed. This is 5 half-dozens.

1999 Q18

Problems 17, 18, and 19 refer to the following:

Cookies For a Crowd



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$1\frac{1}{2}$ cups of flour, 2 eggs, 3 tablespoons butter, $\frac{3}{4}$ cups sugar, and 1 package of chocolate drops. They will make only full recipes, not partial recipe.

18. They learn that a big concert is scheduled for the same night and attendance will be down 25%. How many recipes of cookies should they make for their smaller party?

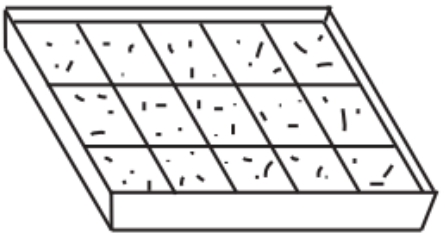
(A) 6 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

18. **Answer (E):** The $108(0.75) = 81$ students need 2 cookies each so 162 cookies are to be baked. Since $162 \div 15 = 10.8$, Walter and Gretel must bake 11 recipes. A few leftovers are a good thing!

1999 Q19

Problems 17, 18, and 19 refer to the following:

Cookies For a Crowd



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$1\frac{1}{2}$ cups of flour, 2 eggs, 3 tablespoons butter, $\frac{3}{4}$ cups sugar, and 1 package of chocolate drops. They will make only full recipes, not partial recipe.

19. The drummer gets sick. The concert is cancelled. Walter and Gretel must make enough pans of cookies to supply 216 cookies. There are 8 tablespoons in a stick of butter. How many sticks of butter will be needed? (Some butter may be left over, of course.)

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

19. **Answer (B):** Since $216 \div 15 = 14.4$, they will have to bake 15 recipes. This requires $15 \times 3 = 45$ tablespoons of butter. So, $45 \div 8 = 5.625$, and 6 sticks are needed.