2002 Q17

17. In a mathematics contest with ten problems, a student gains 5 points for a correct answer and loses 2 points for an incorrect answer. If Olivia answered every problem and her score was 29, how many correct answers did she have?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

17. **(C)** Olivia solved at least 6 correctly to have scored over 25. Her score for six correct would be $6(+5)+4(-2)=22$, which is too low. If she answered 7 correctly, her score would be $7(+5)+3(-2)=29$, and this was her score. The correct choice is (C).

OR

Make a table:

Right	+ score	Wrong	- score	Total
9	45	1	-2	43
8	40	2	-4	36
7	35	3	-6	29

OR

If Olivia answers x problems correctly, then she answered $10-x$ incorrectly and her score was

$$\begin{aligned} 5x-2(10-x) &= 29 \\ 5x-20+2x &= 29 \\ 7x &= 49 \\ x &= 7 \end{aligned}$$

2000 Q17

17. The operation $\otimes$ is defined for all nonzero numbers by $a \otimes b = \frac{a^2}{b}$. Determine $[(1 \otimes 2) \otimes 3] - [1 \otimes (2 \otimes 3)]$.

(A) $-\frac{2}{3}$ **(B)** $-\frac{1}{4}$ **(C)** 0 **(D)** $\frac{1}{4}$ **(E)** $\frac{2}{3}$

17. **Answer (A):** We have

$$(1 \otimes 2) \otimes 3 = \frac{1^2}{2} \otimes 3 = \frac{1}{2} \otimes 3 = \frac{(\frac{1}{2})^2}{3} = \frac{\frac{1}{4}}{3} = \frac{1}{12},$$

and

$$1 \otimes (2 \otimes 3) = 1 \otimes (\frac{2^2}{3}) = 1 \otimes \frac{4}{3} = \frac{1^2}{\frac{4}{3}} = \frac{3}{4}.$$

Therefore,

$$(1 \otimes 2) \otimes 3 - 1 \otimes (2 \otimes 3) = \frac{1}{12} - \frac{3}{4} = \frac{1}{12} - \frac{9}{12} = -\frac{8}{12} = -\frac{2}{3}.$$

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2013 Q17

17. The sum of six consecutive positive integers is 2013. What is the largest of these six integers?

(A) 335 **(B)** 338 **(C)** 340 **(D)** 345 **(E)** 350

17. **Answer (B):** The average of the six integers is $\frac{2013}{6} = 335.5$, so $2013 = 333 + 334 + 335 + 336 + 337 + 338$. The largest of the six integers is 338.

2017 Q17

17. Starting with some gold coins and some empty treasure chests, I tried to put 9 gold coins in each treasure chest, but that left 2 treasure chests empty. So instead I put 6 gold coins in each treasure chest, but then I had 3 gold coins left over. How many gold coins did I have?

(A) 9 **(B)** 27 **(C)** 45 **(D)** 63 **(E)** 81

17. **Answer (C):** After putting 9 coins in all but two of the chests, I could take 3 coins out of each chest to leave 6 coins in those chests. Doing this would allow me to fill the remaining 2 chests with 6 coins each, and have another 3 coins left over. So I must have removed 15 coins (3 from each of 5 chests). Thus I initially had put 9 coins into each of 5 chests, making for a total of 45 coins (and 7 chests).

OR

Let c be the number of chests. Then $9(c-2) = 6c + 3$. Solving yields $c = 7$, so the number of coins is $6 \cdot 7 + 3 = 9(7-2) = 45$.

2012 Q19

19. In a jar of red, green, and blue marbles, all but 6 are red marbles, all but 8 are green, and all but 4 are blue. How many marbles are in the jar?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 18

19. **Answer (C):** There are 4 non-blue marbles. That is, there are altogether 4 red and green marbles. There are also 6 non-red marbles and 8 non-green marbles, so there are two more red marbles than green marbles. Therefore there are 3 red marbles and 1 green marble. Because 3 of the 8 non-green marbles are red, the other 5 must be blue. The total number of marbles is $1 + 3 + 5 = 9$.

OR

Let g, b, and r be the number of green, blue, and red marbles respectively. Then

$$g + b = 6$$

$$r + b = 8$$

$$r + g = 4$$

Adding all three equations together, $2g + 2b + 2r = 18$, so $g + b + r = 9$.