2005 Q12

12. Big Al, the ape, ate 100 bananas from May 1 through May 5. Each day he ate six more bananas than on the previous day. How many bananas did Big Al eat on May 5?

(A) 20 **(B)** 22 **(C)** 30 **(D)** 32 **(E)** 34

12. **(D)** You can solve this problem by guessing and checking. If Big Al had eaten 10 bananas on May 1, then he would have eaten $10 + 16 + 22 + 28 + 34 = 110$ bananas. This is 10 bananas too many, so he actually ate 2 fewer bananas each day. Thus, Big Al ate 8 bananas on May 1 and 32 bananas on May 5.

OR

The average number of bananas eaten per day was $\frac{100}{5} = 20$. Because the number of bananas eaten on consecutive days differs by 6 and there are an odd number of days, the average is also the median. Therefore, the average number of bananas he ate per day, 20, is equal to the number of bananas he ate on May 3. So on May 5 Big Al ate $20 + 12 = 32$ bananas.

OR

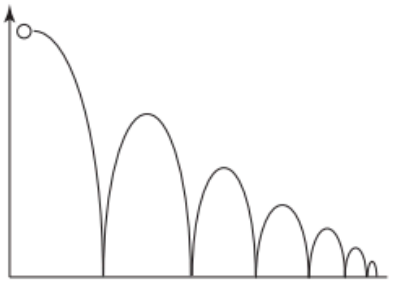
Let x be the number of bananas that Big Al ate on May 5. The following chart documents his banana intake for the five days.

May 5	May 4	May 3	May 2	May 1
x	$x-6$	$x-12$	$x-18$	$x-24$

The total number of bananas Big Al ate was $5x - 60$, which must be 100. So Big Al ate $x = \frac{160}{5} = 32$ bananas on May 5.

2008 Q12

12. A ball is dropped from a height of 3 meters. On its first bounce it rises to a height of 2 meters. It keeps falling and bouncing to $\frac{2}{3}$ of the height it reached in the previous bounce. On which bounce will it not rise to a height of 0.5 meters?



(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

12. **Answer (C):** The table gives the height of each bounce.

Bounce	1	2	3	4	5
Height in Meters	2	$\frac{2}{3} \cdot 2 = \frac{4}{3}$	$\frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$	$\frac{2}{3} \cdot \frac{8}{9} = \frac{16}{27}$	$\frac{2}{3} \cdot \frac{16}{27} = \frac{32}{81}$

Because $\frac{16}{27} > \frac{16}{32} = \frac{1}{2}$ and $\frac{32}{81} < \frac{32}{64} = \frac{1}{2}$, the ball first rises to less than 0.5 meters on the fifth bounce.

Note: Because all the fractions have odd denominators, it is easier to double the numerators than to halve the denominators. So compare $\frac{16}{27}$ and $\frac{32}{81}$ to their numerators' fractional equivalents of $\frac{1}{2}$, $\frac{16}{32}$ and $\frac{32}{64}$.

1992 Q15

15. What is the 1992nd letter in this sequence?

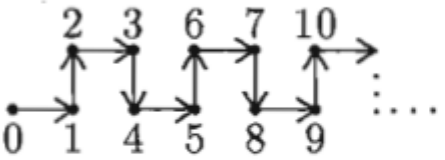
ABCDEDCBAABCDEDCBAABCDEDCBAABCDEDC···

(A) A (B) B (C) C (D) D (E) E

15. (C) The pattern repeats every 9 letters. Dividing 1992 by 9 yields a remainder of 3. Therefore, the 1992nd letter corresponds to the third letter in the sequence, which is C.

1994 Q15

15. If this path is to continue in the same pattern:



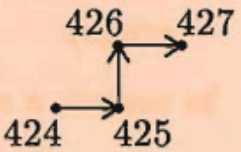
then which sequence of arrows goes from point 425 to point 427?

(A)  (B)  (C)  (D)  (E) 

15. (A) The path repeats every four numbers. Since 425 leaves a remainder of 1 when divided by 4 and 427 leaves a remainder of 3 when divided by 4, it follows that the pattern from point 425 to point 427 is the same as that from point 1 to point 3.

OR

The pattern starts over every four numbers, that is, at 0, 4, 8, etc. All the multiples of 4 are in the same position in their section of the pattern, so draw the portion of the diagram using the multiple of 4 just less than 425.



1998 Q15

Problems 15, 16, 17 all refer to the following:

Don't Crowd The Isles

In the very center of the Irenic Sea lie the beautiful Nisos Isles. In 1998 the number of people on these islands is only 200, but the population triples every 25 years. Queen Irene has decreed that there must be at least 1.5 square miles for every person living in the Isles. The total area of the Nisos Isles is 24,900 square miles.

15. Estimate the population of Nisos in the year 2050 .

(A) 600 **(B)** 800 **(C)** 1000 **(D)** 2000 **(E)** 3000

15. **Answer (D):**
In the year 1998 the population is 200.
In 2023 the population will be $200(3) = 600$.
In 2048 the population will be $600(3) = 1800$.
In 2050 the population will be about 2000.

1998 Q16

Problems 15, 16, 17 all refer to the following:

Don't Crowd The Isles

In the very center of the Irenic Sea lie the beautiful Nisos Isles. In 1998 the number of people on these islands is only 200, but the population triples every 25 years. Queen Irene has decreed that there must be at least 1.5 square miles for every person living in the Isles. The total area of the Nisos Isles is 24,900 square miles.

16. Estimate the year in which the population of Nisos will be approximately 6,000 .

(A) 2050 **(B)** 2075 **(C)** 2100 **(D)** 2125 **(E)** 2150

16. **Answer (B):**

Year	Population
1998	200
2023	600
2048	1,800
2073	5,400
2098	16,200

Of the choices available the year 2075 is the best estimate.

1998 Q17

Problems 15, 16, 17 all refer to the following:

Don't Crowd The Isles

In the very center of the Irenic Sea lie the beautiful Nisos Isles. In 1998 the number of people on these islands is only 200, but the population triples every 25 years. Queen Irene has decreed that there must be at least 1.5 square miles for every person living in the Isles. The total area of the Nisos Isles is 24,900 square miles.

17. In how many years, approximately, from 1998 will the population of Nisos be as much as Queen Irene has proclaimed that the islands can support?

(A) 50 yrs. **(B)** 75 yrs. **(C)** 100 yrs. **(D)** 125 yrs. **(E)** 150 yrs.

17. **Answer (C):**

Year	Population	Area Needed
1998	200	300
2023	600	900
2048	1,800	2,700
2073	5,400	8,100
2098	16,200	24,300

The Isles can support $24{,}900 \div 1.5 = 16{,}600$ people. The chart shows that this will happen about the year 2098, or in about 100 years.