2008 Q11

11. Each of the 39 students in the eighth grade at Lincoln Middle School has one dog or one cat or both a dog and a cat. Twenty students have a dog and 26 students have a cat. How many students have both a dog and a cat?

(A) 7　　　(B) 13　　　(C) 19　　　(D) 39　　　(E) 46

11. **Answer (A):** The number of cat owners plus the number of dog owners is $20 + 26 = 46$. Because there are only 39 students in the class, there are $46 - 39 = 7$ students who have both.

<div align="center">OR</div>

Because each student has at least a cat or a dog, there are $39 - 20 = 19$ students with a cat but no dog, and $39 - 26 = 13$ students with a dog but no cat. So there are $39 - 13 - 19 = 7$ students with both a cat and a dog.
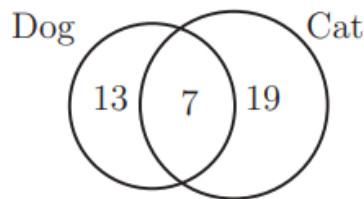
2007 Q13

13. Sets A and B, shown in the Venn diagram, have the same number of elements. Their union has 2007 elements and their intersection has 1001 elements. Find the number of elements in A.
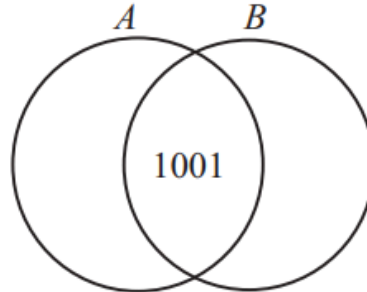
(A) 503   (B) 1006   (C) 1504   (D) 1507   (E) 1510

13. **(C)** Let C denote the set of elements that are in A but not in B. Let D denote the set of elements that are in B but not in A. Because sets A and B have the same number of elements, the number of elements in C is the same as the number of elements in D. This number is half the number of elements in the union of A and B minus the intersection of A and B. That is, the number of elements in each of C and D is

$$\frac{1}{2}(2007 - 1001) = \frac{1}{2} \cdot 1006 = 503.$$

Adding the number of elements in A and B to the number in A but not in B gives $1001 + 503 = 1504$ elements in A.
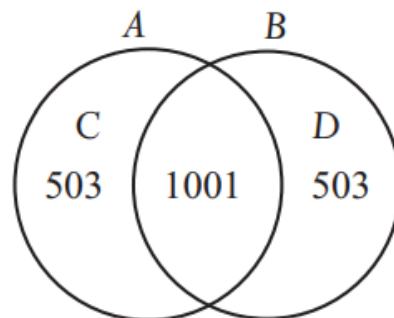

OR

Let x be the number of elements each in A and B. Then $2x - 1001 = 2007$, $2x = 3008$ and $x = 1504$.

2

1998 Q14

14. At Annville Junior High School, 30% of the students in the Math Club are in the Science Club, and 80% of the students in the Science Club are in the Math Club. There are 15 students in the Science Club. How many students are in the Math Club?

(A) 12   (B) 15   (C) 30   (D) 36   (E) 40

14. **Answer (E):** Since 80% of the Science Club members are also in the Math Club, there are $0.8(15) = 12$ students common to both clubs. Because 30% of the students in the Math Club are also in the Science Club, there are $12 \div 0.3 = 40$ students in the Math Club.

Math   Science

28 (12) 3

28+12=40
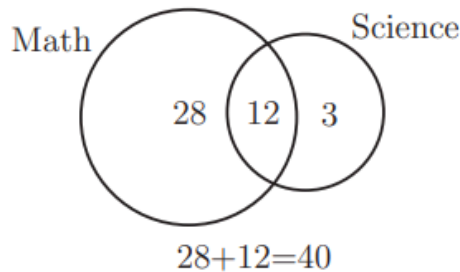
2015 Q15

15. At Euler Middle School, 198 students voted on two issues in a school referendum with the following results: 149 voted in favor of the first issue and 119 voted in favor of the second issue. If there were exactly 29 students who voted against both issues, how many students voted in favor of both issues?

(A) 49   (B) 70   (C) 79   (D) 99   (E) 149

15. **Answer (D):** The sum $149 + 119 + 29 = 297$ counts the number of students who voted for both issues twice. So the number who voted in favor of both issues is $297 - 198$ or 99.

OR

In the diagram below, the left circle represents the 149 students who voted for the first issue, and the right circle represents the 119 students who voted for the second issue. Let x be the number of students who voted for both issues. Then $149 - x$ students voted for the first issue but not the second, $119 - x$ students

voted for the second issue but not the first and 29 students voted against both issues. The sum of the numbers in the diagram must be 198, so

$$(149 - x) + x + (119 - x) + 29 = 198,$$
$$297 - x = 198,$$
$$x = 99.$$