2014 Q24

24. One day the Beverage Barn sold 252 cans of soda to 100 customers, and every customer bought at least one can of soda. What is the maximum possible median number of cans of soda bought per customer on that day?

(A) 2.5 **(B)** 3.0 **(C)** 3.5 **(D)** 4.0 **(E)** 4.5

24. **Answer (C):** Suppose the numbers of cans purchased by the 100 customers are listed in increasing order. The median is the average of the 50th and 51st numbers in the ordered list. To maximize the median, minimize the first 49 numbers by taking them all to be 1. If the 50th number is 4, then the sum of all 100 numbers would at least $49 + 51 \cdot 4 = 253$, which is too large. If instead the 50th number is 3 and the following numbers all equal 4, then the sum of the 100 numbers is $49 + 3 + 50 \cdot 4 = 252$ and the median is $(3 + 4) \div 2 = 3.5$.

OR

To maximize the median, the largest 50 should be the same and as large as possible. The lower 49 should be as small as possible. The median of the list will be the average of the 50th and 51st numbers. If every customer has 1 can of soda, there are 152 left to distribute. Giving the upper 50 three more each gives the top 50 four cans each (200 total) and the lower 50 one each (50 total). There are 2 cans left. Giving the 50th person the extra 2 means the 50th has 3 cans, and the 51st has 4 cans for a median of $(3 + 4) \div 2 = 3.5$.

1986 Q24

24. The 600 students at King Middle School are divided into three groups of equal size for lunch. Each group has lunch at a different time. A computer randomly assigns each student to one of the three lunch groups. The probability that three friends, Al, Bob, and Carol, will be assigned to the same lunch group is approximately

A) $\frac{1}{27}$ B) $\frac{1}{9}$ C) $\frac{1}{8}$ D) $\frac{1}{6}$ E) $\frac{1}{3}$

24. (B) Al must be assigned to one of the lunch groups. The probability that Bob is assigned to the same lunch group is approximately $\frac{1}{3}$ ($\frac{199}{599}$ exactly) and the probability that Carol is assigned to that same group is also approximately $\frac{1}{3}$ ($\frac{198}{598}$). Thus the probability that all three are assigned to the same group is approximately $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

2007 Q24

24. A bag contains four pieces of paper, each labeled with one of the digits 1, 2, 3 or 4, with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of 3?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

24. **(C)** A number is a multiple of three when the sum of its digits is a multiple of 3. If the number has three distinct digits drawn from the set $\{1, 2, 3, 4\}$, then the sum of the digits will be a multiple of three when the digits are $\{1, 2, 3\}$ or $\{2, 3, 4\}$. That means the number formed is a multiple of three when, after the

three draws, the number remaining in the bag is 1 or 4. The probability of this occurring is $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

2008 Q24

24. Ten tiles numbered 1 through 10 are turned face down. One tile is turned up at random, and a die is rolled. What is the probability that the product of the numbers on the tile and the die will be a square?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{11}{60}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{7}{30}$

24. **Answer (C):** There are $10 \times 6 = 60$ possible pairs. The squares less than 60 are 1, 4, 9, 16, 25, 36 and 49. The possible pairs with products equal to the given squares are $(1,1)$, $(2,2)$, $(1,4)$, $(4,1)$, $(3,3)$, $(9,1)$, $(4,4)$, $(8,2)$, $(5,5)$, $(6,6)$ and $(9,4)$. So the probability is $\frac{11}{60}$.

1987 Q24

24. A multiple choice examination consists of 20 questions. The scoring is +5 for each correct answer, −2 for each incorrect answer, and 0 for each unanswered question. John's score on the examination is 48. What is the maximum number of questions he could have answered correctly?

A) 9 B) 10 C) 11 D) 12 E) 16

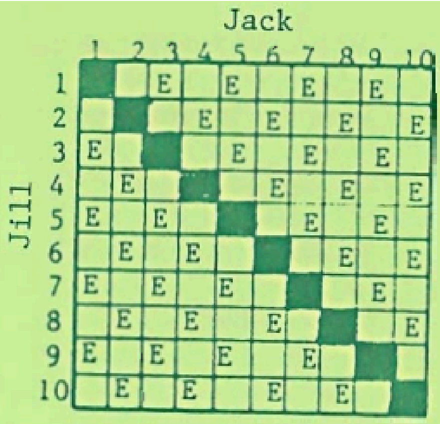
24. D If John answered 13 or more questions correctly, then his score would have been at least 13(5) - 7(2) = 51 (13 correct, 7 incorrect). Checking the other cases, we find that John could have answered 12 correctly, 6 incorrectly, and left 2 unanswered for a score of 12(5) - 6(2) = 48. Note that 10 correct, 1 incorrect, 9 unanswered also give a score of 48.

1987 Q25

25. Ten balls numbered 1 to 10 are in a jar. Jack reaches into the jar and randomly removes one of the balls. Then Jill reaches into the jar and randomly removes a different ball. The probability that the sum of the two numbers on the balls removed is even is

A) $\frac{4}{9}$ B) $\frac{9}{19}$ C) $\frac{1}{2}$ D) $\frac{10}{19}$ E) $\frac{5}{9}$

25. A Since Jack and Jill cannot remove the same number, there are $10 \cdot 9 = 90$ ways they can remove the two balls from the jar as shown by the unshaded squares on the grid. Those squares representing an even sum are labeled "E". There are 40 such squares - 4 in each column (or row) since the two numbers must both be odd or both be even. The probability is $\frac{40}{90} = \frac{4}{9}$.



OR

There are 10 ways to select the first number but only 4 ways to select the second since it must have the same parity (both odd or both even) as the first. Thus the probability is $\frac{10 \cdot 4}{10 \cdot 9} = \frac{4}{9}$.

1986 Q25

25. Which of the following sets of whole numbers has the largest average?

A) multiples of 2 between 1 and 101
B) multiples of 3 between 1 and 101
C) multiples of 4 between 1 and 101
D) multiples of 5 between 1 and 101
E) multiples of 6 between 1 and 101

25. (D) In a set of whole numbers which are equally spaced, the average of the numbers in the set is the average of the smallest number and the largest number. For example, the average of { 1,3,5,7,9 } = $\frac{1+9}{2} = 5$, and the average of { 2,5,8,11,14,17 } is $\frac{2+17}{2} = 9.5$.

In this problem, then, the averages are:

A: $\frac{2+100}{2} = 51$, B: $\frac{3+99}{2} = 51$,

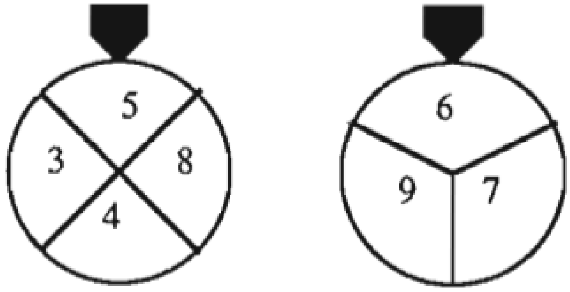
C: $\frac{4+100}{2} = 52$, D: $\frac{5+100}{2} = 52.5$,

E: $\frac{6+96}{2} = 51$.

One could "guesstimate" that the set with the "largest" numbers should have the largest average. The numbers 5 and 100 are (overall) larger than the corresponding numbers in the other sets.

1989 Q25

25. Every time these two wheels are spun, two numbers are selected by the pointers. What is the probability that the sum of the two selected numbers is even?



A) $\frac{1}{6}$ B) $\frac{3}{7}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) $\frac{5}{7}$

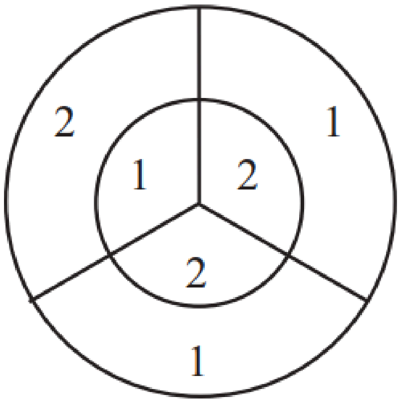
25. C The set of all the possible outcomes of spinning each wheel is
{ (5, 6) . (5, 7), (5, 9), (8,6), (8, 7), (8,9), (4,6), (4,7), (4,9), (3,6), (3,7), (3,9) }
There are six even sums among the 12 outcomes for a probability of $\frac{6}{12} = \frac{1}{2}$.

OR

Consider the outcome on the right wheel first. Whatever number occurs, there are two out of four chances for the number on the left wheel to give an even sum. Thus the desired probability is $\frac{2}{4} = \frac{1}{2}$.

2007 Q25

25. On the dart board shown in the figure, the outer circle has radius 6 and the inner circle has radius 3. Three radii divide each circle into three congruent regions, with point values shown. The probability that a dart will hit a given region is proportional to the area of the region. When two darts hit this board, the score is the sum of the point values in the regions. What is the probability that the score is odd?



(A) $\frac{17}{36}$ **(B)** $\frac{35}{72}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{37}{72}$ **(E)** $\frac{19}{36}$

25. **(B)** The outer circle has area 36π and the inner circle has area 9π, making the area of the outer ring $36\pi - 9\pi = 27\pi$. So each region in the outer ring has area $\frac{27\pi}{3} = 9\pi$, and each region in the inner circle has area $\frac{9\pi}{3} = 3\pi$. The probability of hitting a given region in the inner circle is $\frac{3\pi}{36\pi} = \frac{1}{12}$, and the probability of hitting a given region in the outer ring is $\frac{9\pi}{36\pi} = \frac{1}{4}$. For the score to be odd, one of the numbers must be 1 and the other number must be 2. The probability of hitting a 1 is

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{12} = \frac{7}{12},$$

and the probability of hitting a 2 is

$$1 - \frac{7}{12} = \frac{5}{12}.$$

Therefore, the probability of hitting a 1 and a 2 in either order is

$$\frac{7}{12} \cdot \frac{5}{12} + \frac{5}{12} \cdot \frac{7}{12} = \frac{70}{144} = \frac{35}{72}.$$