1991 Q17

17. An auditorium with 20 rows of seats has 10 seats in the first row. Each successive row has one more seat than the previous row. If students taking an exam are permitted to sit in any row, but not next to another student in that row, then the maximum number of students that can be seated for an exam is

(A) 150 **(B)** 180 **(C)** 200 **(D)** 400 **(E)** 460

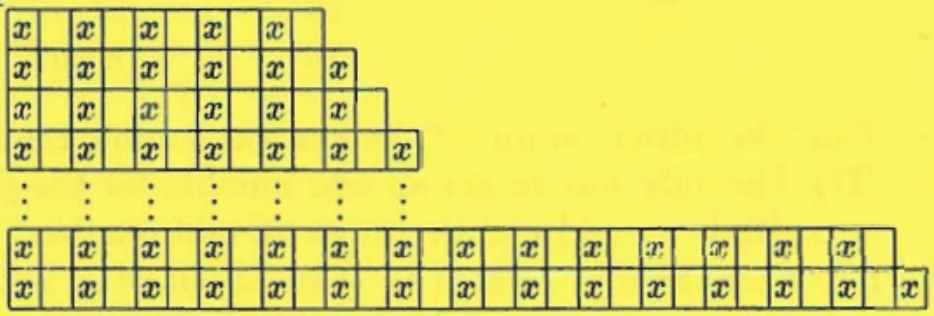
17. **(C)** The first row has 10 seats, so 5 students can sit in row 1. The second row has 11 seats, so 6 students can sit in row 2. The third row has 12 seats, so 6 students can sit in row 3. ... The last (20^{th}) row has 29 seats, so 15 students can sit in row 20. The sum is $5+6+6+7+7+\cdots+14+14+15$. Regrouping yields

$$\begin{array}{rr} & 5+\ 6+\ 6+\ 7+\ 7+\ 8+\ 8+\ 9+\ 9+10 \\ +\ & 15+14+14+13+13+12+12+11+11+10 \\ \hline = & 20+20+20+20+20+20+20+20+20+20 \end{array} = 10(20) = 200.$$

OR

Draw a diagram:

Row	Seats	Students
1	10	5
2	11	6
3	12	6
4	13	7
⋮	⋮	⋮
19	28	14
20	29	15



Thus, the sum is $5+6+6+7+7+\cdots+14+14+15 = 200$.

2015 Q18

18. An arithmetic sequence is a sequence in which each term after the first is obtained by adding a constant to the previous term. For example, 2, 5, 8, 11, 14 is an arithmetic sequence with five terms, in which the first term is 2 and the constant added is 3. Each row and each column in this 5×5 array is an arithmetic sequence with five terms. What is the value of X?

(A) 21 **(B)** 31 **(C)** 36 **(D)** 40 **(E)** 42

1				25
		X		
17				81

18. **Answer (B):** The middle number in an arithmetic sequence with 5 terms is the average of the first and last numbers. The average of 1 and 25 is 13. The average of 17 and 81 is 49. Thus, X is the average of 13 and 49, or 31. Alternatively, $X = \frac{9+53}{2} = 31$. In fact X is the average of the four corner entries.

1		13		25
9		31		53
17		49		81

19. What is the 100th number in the arithmetic sequence:
1, 5, 9, 13, 17, 21, 25, ... ?

A) 397 B) 399 C) 401 D) 403 E) 405

1988 Q19

19. A Adding 3 to each term in the original arithmetic sequence yields the sequence 4,8,12,16,20, ... in which the one-hundredth term is 400. Subtracting 3 from each term shows that 397 is the one-hundredth term of the original sequence.

OR

We may obtain the 100th term in the sequence by adding 4 to the first term 1 a total of 99 times. Thus the 100th term is $1 + 99(4) = 397$.

1992 Q19

19. The distance between the 5^{th} and 26^{th} exits on an interstate highway is 118 miles. If any two exits are at least 5 miles apart, then what is the largest number of miles there can be between two consecutive exits that are between the 5^{th} and 26^{th} exits?

(A) 8 (B) 13 (C) 18 (D) 47 (E) 98

19. (C) There are 21 segments between the 5^{th} and 26^{th} exits. Using the minimum length of 5 miles, 20 segments would yield $20 \times 5 = 100$ miles. This leaves $118 - 100 = 18$ miles for the other segment.

OR

There are 21 segments between the 5^{th} and 26^{th} exits. If each segment were its minimal 5 miles length, then the total distance between the 5^{th} and 26^{th} exits would be 105 miles. Since $118 - 105 = 13$, all 13 additional miles could occur between one pair of consecutive exits. Such a pair would be $5 + 13 = 18$ miles apart.