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## 1992 Q6

6. Suppose that  $\begin{array}{c} \triangle \\ a \\ b \quad c \end{array}$  means  $a + b - c$ . For example,  $\begin{array}{c} \triangle \\ 5 \\ 4 \quad 6 \end{array}$  is  $5 + 4 - 6 = 3$ .

Then the sum  $\begin{array}{c} \triangle \\ 1 \\ 3 \quad 4 \end{array} + \begin{array}{c} \triangle \\ 2 \\ 5 \quad 6 \end{array}$  is

- (A) -2      (B) -1      (C) 0      (D) 1      (E) 2

6. (D)  $(1 + 3 - 4) + (2 + 5 - 6) = 0 + 1 = 1$ .

OR

Note that  $\begin{array}{c} \triangle \\ a \\ b \quad c \end{array} + \begin{array}{c} \triangle \\ d \\ e \quad f \end{array} = \begin{array}{c} \triangle \\ x \\ y \quad z \end{array}$  where  $x = a + d$ ,  $y = b + e$  and  $z = c + f$ ; i.e., the sum of two 'triangular expressions' is the value of the 'triangular expression' obtained by summing the respective components. It follows that the required sum is  $(1 + 2) + (3 + 5) - (4 + 6) = 3 + 8 - 10 = 1$ .

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## 1993 Q6

6. A can of soup can feed 3 adults or 5 children. If there are 5 cans of soup and 15 children are fed, then how many adults would the remaining soup feed?

- (A) 5      (B) 6      (C) 7      (D) 8      (E) 10

6. (B) Three cans of soup are needed for 15 children, so the remaining 2 cans of soup will feed  $2 \times 3 = 6$  adults.

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## 1994 Q6

6. The unit's digit (one's digit) of the product of any six consecutive positive whole numbers is
- (A) 0    (B) 2    (C) 4    (D) 6    (E) 8

6. (A) Any selection of six consecutive positive whole numbers has at least one multiple of 5 and at least one multiple of 2. Thus, the product has at least one factor of  $5 \times 2 = 10$  and must end in 0.

OR

The statement of the problem implies that any six consecutive positive numbers can be used. Compute  $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 120$  to find that the last digit is 0. Or, even easier, multiply any six consecutive integers containing 10 to see without multiplication that the last digit must be 0.

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## 1996 Q6

6. What is the smallest result that can be obtained by the following process?
- Choose three different numbers from the set  $\{3, 5, 7, 11, 13, 17\}$ .
  - Add two of these numbers.
  - Multiply their sum by the third number.
- (A) 15    (B) 30    (C) 36    (D) 50    (E) 56

6. (C) To obtain the smallest result, use the three smallest numbers. This yields three choices:

$$3(5 + 7) = 36, \quad 5(3 + 7) = 50, \quad \text{and} \quad 7(3 + 5) = 56.$$

Thus 36 is the smallest result.

**OR**

Since multiplication is repeated addition, it follows that the smallest result should use the smallest number as the multiplier and the other two of the three smallest numbers for the sum. Thus  $3(5 + 7) = 36$  is the smallest result.

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**1996 Q7**

7. Brent has goldfish that quadruple (become four times as many) every month, and Gretel has goldfish that double every month. If Brent has 4 goldfish at the same time that Gretel has 128 goldfish, then in how many months from that time will they have the same number of goldfish?
- (A) 4    (B) 5    (C) 6    (D) 7    (E) 8

7. (B) Make a table.

Month:	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
Brent:	4	16	64	256	1024	4096
Gretel:	128	256	512	1024	2048	4096

Thus it takes 5 months for the number of goldfish to be equal.

OR

The ratio of the number of Gretel's goldfish to the number of Brent's goldfish decreases by a factor of 2 every month. Because they initially differ by a factor of  $128/4 = 32 = 2^5$ , this indicates that the process will take 5 months.

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7. How many whole numbers are between  $\sqrt{8}$  and  $\sqrt{80}$  ?

- A) 5    B) 6    C) 7    D) 8    E) 9

1986 Q7

7. (B) Since  $\sqrt{8} < \sqrt{9} = 3$ , and  $\sqrt{80} < \sqrt{81} = 9$ , the desired whole numbers are 3,4,5,6,7,8; there are six of them.

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## 1985 Q8

8. If  $a = -2$ , the largest number in the set  $\{-3a, 4a, \frac{24}{a}, a^2, 1\}$  is
- A)  $-3a$       B)  $4a$       C)  $\frac{24}{a}$       D)  $a^2$       E)  $1$

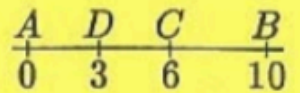
8. (A) If  $a = -2$ , the set is  $\{6, -8, -12, 4, 1\}$  so  $6 = 3a$  is the largest. Notice that  $4a$  and  $\frac{24}{a}$  could be eliminated immediately since they are negative if  $a$  is negative.

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## 1996 Q8

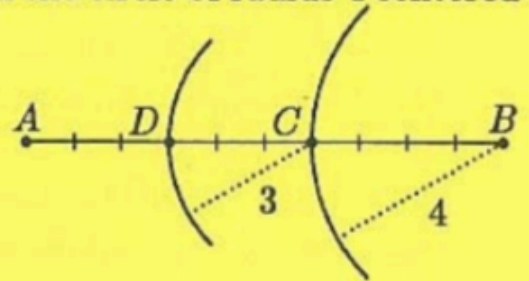
8. Points  $A$  and  $B$  are 10 units apart. Points  $B$  and  $C$  are 4 units apart. Points  $C$  and  $D$  are 3 units apart. If  $A$  and  $D$  are as close as possible, then the number of units between them is
- (A) 0      (B) 3      (C) 9      (D) 11      (E) 17

8. (B) The shortest distance occurs when the four points are collinear. Then  $BC = 4$  and  $CD = 3$  so that  $AD = 3$  as shown.



OR

Once  $A$  and  $B$  are chosen,  $C$  can be anywhere on the circle of radius 4 centered at  $B$ . The closest point to  $A$  would be on the segment between  $B$  and  $A$ . Then  $D$  could be anywhere on the circle of radius 3 centered at  $C$ . Once again the closest point to  $A$  is on the line segment between  $C$  and  $A$ . Thus the distance from  $A$  to  $D$  is 3 units.



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9. Consider the operation  $*$  defined by the following table:

$*$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

For example,  $3 * 2 = 1$ . Then  $(2 * 4) * (1 * 3) =$

1993 Q9

- (A) 1      (B) 2      (C) 3      (D) 4      (E) 5

9. (D) Substituting the values from the table yields

$$(2 * 4) * (1 * 3) = 3 * 3 = 4.$$

**Query.** Would evaluating the products

$((2 * 4) * 1) * 3$ ,  $(2 * (4 * 1)) * 3$ ,  $2 * ((4 * 1)) * 3$  and  $2 * (4 * (1 * 3))$ ,  
yield the same result?

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1996 Q9

9. If 5 times a number is 2, then 100 times the reciprocal of the number is  
(A) 2.5    (B) 40    (C) 50    (D) 250    (E) 500

9. (D) To find the number, divide 2 by 5 to obtain  $2/5$ . The reciprocal of  $2/5$  is  $5/2$ , and 100 times  $5/2$  equals 250.

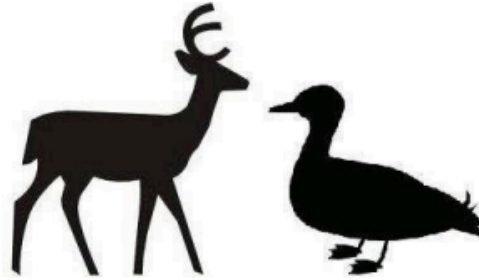
OR

Let  $n$  be the unknown number. Then  $5 \cdot n = 2$  so  $n = 2/5$ . The reciprocal of  $2/5$  is  $5/2$ , and  $100 \cdot 5/2 = 250$ .

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9. The Fort Worth Zoo has a number of two-legged birds and a number of four-legged mammals. On one visit to the zoo, Margie counted 200 heads and 522 legs. How many of the animals that Margie counted were two-legged birds?

(A) 61      (B) 122      (C) 139      (D) 150      (E) 161



9. **Answer (C):** All 200 heads belonged to animals with at least two legs, accounting for 400 of the 522 legs. The additional 122 legs belonged to four-legged mammals, each of which had two additional legs. So Margie saw  $\frac{122}{2} = 61$  four-legged mammals and  $200 - 61 = 139$  birds.

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2016 Q10

10. Suppose that  $a * b$  means  $3a - b$ . What is the value of  $x$  if

$$2 * (5 * x) = 1 ?$$

(A)  $\frac{1}{10}$       (B) 2      (C)  $\frac{10}{3}$       (D) 10      (E) 14



10. Answer (D):

Since  $2 * (5 * x) = 1$ , it follows that  $6 - (5 * x) = 1$ , and so  $5 * x = 5$ . Applying the formula again,  $15 - x = 5$ , and therefore  $x = 10$ .