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2001 Q21

21. The mean of a set of five different positive integers is 15. The median is 18. The maximum possible value of the largest of these five integers is

(A) 19 (B) 24 (C) 32 (D) 35 (E) 40

21. (D) The sum of all five numbers is 5(15)=75. Let the numbers be $W, X, 18, Y$ and Z in increasing order. For Z to be as large as possible, make W, X and Y as small as possible. The smallest possible values are $W = 1, X = 2$ and $Y = 19$. Then the sum of $W, X, 18$ and Y is 40, and the difference, $75 - 40 = 35$, is the largest possible value of Z.

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2007 Q21

21. Two cards are dealt from a deck of four red cards labeled A, B, C, D and four green cards labeled A, B, C, D. A winning pair is two of the same color or two of the same letter. What is the probability of drawing a winning pair?

(A) $\frac{2}{7}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{4}{7}$ **(E)** $\frac{5}{8}$

21. **(D)** After the first card is dealt, there are seven left. The three cards with the same color as the initial card are winners and so is the card with the same letter but a different color. That means four of the remaining seven cards form winning pairs with the first card, so the probability of winning is $\frac{4}{7}$.

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1989 Q21

21. Jack had a bag of 128 apples. He sold 25% of them to Jill. Next he sold 25% of those remaining to June. Of those apples still in his bag, he gave the shiniest one to his teacher. How many apples did Jack have then?

A) 7 B) 63 C) 65 D) 71 E) 111

21. D Since 25% = .25 = $\frac{1}{4}$, Jill bought $\frac{1}{4}$ x 128 = 32 apples. June bought $\frac{1}{4}$ of the remaining 128 - 32 = 96 apples or 24 apples. This leaves 72 apples -- one for the teacher and 71 for Jack.

OR

Keep track of the number of apples that Jack has at the end of each transaction:

Jill: $\frac{3}{4}$ x 128 = 96

June: $\frac{3}{4}$ x 96 = 72

Teacher: 72 – 1 = 71.

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2000 Q21

21. Keiko tosses one penny and Ephraim tosses two pennies. The probability that Ephraim gets the same number of heads that Keiko gets is

(A) $\frac{1}{4}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

21. **Answer (B):** Make a complete list of equally likely outcomes:

Keiko	Ephraim	Same Number of Heads?
H	HH	No
H	HT	Yes
H	TH	Yes
H	TT	No
T	HH	No
T	HT	No
T	TH	No
T	TT	Yes

The probability that they have the same number of heads is $\frac{3}{8}$.

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2002 Q21

21. Harold tosses a nickel four times. The probability that he gets at least as many heads as tails is

(A) $\frac{5}{16}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{5}{8}$ **(E)** $\frac{11}{16}$

21. **(E)** There are 16 possible outcomes: $HHHH$, $HHHT$, $HHTH$, $HTHH$, $THHH$, $HHTT$, $HTHT$, $HTTH$, $THTH$, $THHT$, $TTHH$ and $HTTT$, $THTT$, $TTHT$, $TTTH$, $TTTT$. The first eleven have at least as many heads as tails. The probability is $\frac{11}{16}$.

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2016 Q21

21. A box contains 3 red chips and 2 green chips. Chips are drawn randomly, one at a time without replacement, until all 3 of the reds are drawn or until both green chips are drawn. What is the probability that the 3 reds are drawn?

(A) $\frac{3}{10}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{2}{3}$

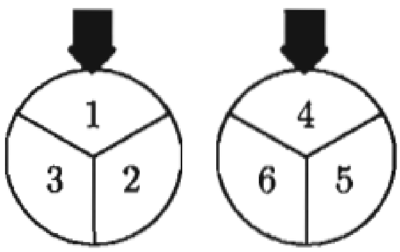
21. Answer (B):

Consider drawing all five chips and listing the 10 possible outcomes: RRRGG, RRGRG, RGRRG, GRRRG, GGRRR, GRGRR, RGGRR, GRRGR, RGRGR, RRGGR.

All 10 of these outcomes are equally likely. The outcomes that end in G correspond to the outcomes where the 3 reds are drawn and the outcomes that end in R correspond to the outcomes where the 2 greens are drawn. The probability that the 3 reds are drawn is $\frac{4}{10} = \frac{2}{5}$.

1991 Q22

22. Each spinner is divided into 3 equal parts. The results obtained from spinning the two spinners are multiplied. What is the probability that this product is an even number?



(A) $\frac{1}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{7}{9}$ **(E)** 1

22. (D) The only way to get an odd number for the product of two numbers is to multiply an odd number times an odd number. This happens if one spins 1 or 3 on the first spinner (2 chances out of 3) and 5 on the second spinner (1 chance out of 3). Thus, the probability of an odd product is $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$. If one does not get an odd product, then the product is even. Hence the probability of an even product is $1 - \frac{2}{9} = \frac{7}{9}$.

OR

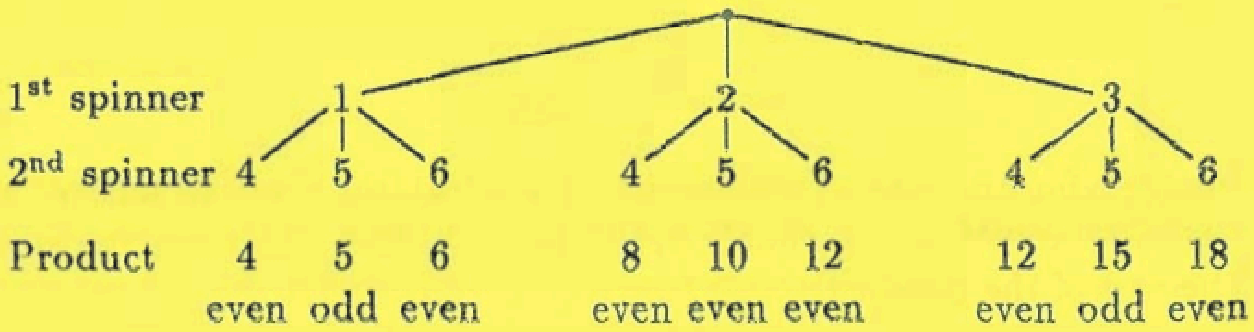
The sample space of pairs to be multiplied is:

$\boxed{1 \times 4}$	$\boxed{2 \times 4}$	$\boxed{3 \times 4}$
1×5	$\boxed{2 \times 5}$	3×5
$\boxed{1 \times 6}$	$\boxed{2 \times 6}$	$\boxed{3 \times 6}$

Successful pairs are marked. The probability is $\frac{7}{9}$.

OR

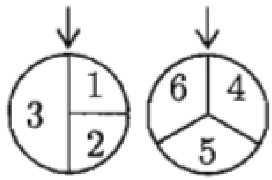
Use a tree diagram:



Thus the probability that the product is even is $\frac{7}{9}$.

1994 Q22

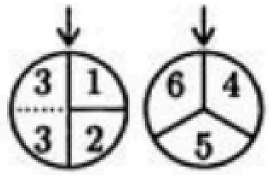
22. The two wheels shown at the right are spun and the two resulting numbers are added. The probability that the sum of the two numbers is even is



(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{5}{12}$ (E) $\frac{4}{9}$

22. **(D)** Subdivide the 3-space on the first wheel so that wheel is divided into four equal regions. Each of the four regions has the same probability of occurring when the first wheel is spun. Hence, the sample space is

(1,4)	(1,5)	(1,6)
(2,4)	(2,5)	(2,6)
(3,4)	(3,5)	(3,6)
(3,4)	(3,5)	(3,6)



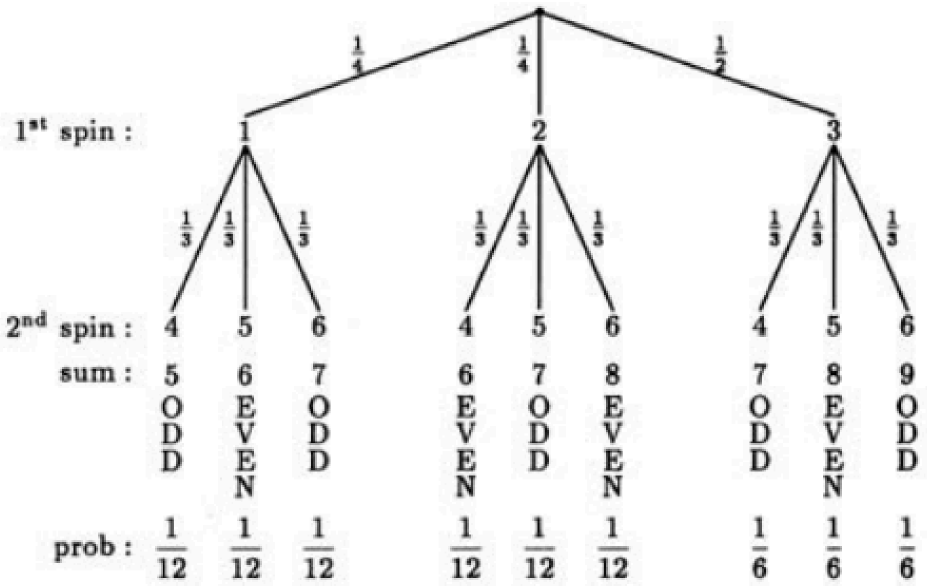
The final three entries were repeated to show the double space for 3 on the first wheel. The sums for these entries are $5, 6, 7, 6, 7, 8, 7, 8, 9, 7, 8, 9$. Five of these twelve are even.

OR

There are two ways to get an even sum, (odd + odd) or (even + even). The first outcome happens if one spins 1 or 3 on the first wheel (3 chances out of 4) and 5 on the second wheel (1 chance out of 3) for a probability of $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$. The second outcome happens if one spins a 2 on the first wheel (1 chance out of 4) and 4 or 6 on the second wheel (2 chances out of 3) for a probability of $\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$. Thus the total probability of an even sum is $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$.

OR

Use a probability tree diagram:



Thus, the probability that the sum is even is $\frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12}$.

OR

List equally likely outcomes in a table. Even sums are marked. The probability is 5/12.

+	4	5	6
1	5	⑥	7
2	⑥	7	⑧
3	7	⑧	9
3	7	⑧	9

OR

$$P(\text{even}) = P(3+5) + P(1+5) + P(2+4) + P(2+6)$$
$$= \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{3}\right) = \frac{5}{12}.$$

1992 Q23

23. If two dice are tossed, the probability that the product of the numbers showing on the tops of the dice is greater than 10 is

(A) $\frac{3}{7}$ (B) $\frac{17}{36}$ (C) $\frac{1}{2}$ (D) $\frac{5}{8}$ (E) $\frac{11}{12}$

23. **(B)** Make a table and fill in the products greater than 10.

×	**1**	**2**	**3**	**4**	**5**	**6**
1 :	1	2	3	4	5	6
2 :	2	4	6	8	10	**12**
3 :	3	6	9	**12**	**15**	**18**
4 :	4	8	**12**	**16**	**20**	**24**
5 :	5	10	**15**	**20**	**25**	**30**
6 :	6	**12**	**18**	**24**	**30**	**36**

Since there are 17 such products out of a possible 36 products, the probability is 17/36.

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2000 Q23

23. There is a list of seven numbers. The average of the first four numbers is 5, and the average of the last four numbers is 8. If the average of all seven numbers is $6\frac{4}{7}$, then the number common to both sets of four numbers is

(A) $5\frac{3}{7}$ (B) 6 (C) $6\frac{4}{7}$ (D) 7 (E) $7\frac{3}{7}$

23. **Answer (B):** Since the average of all seven numbers is $6\frac{4}{7} = \frac{46}{7}$, the sum of the seven numbers is $7 \times \frac{46}{7} = 46$. The sum of the first four numbers is $4 \times 5 = 20$ and the sum of the last four numbers is $4 \times 8 = 32$. Since the fourth number is used in each of these two sums, the fourth number must be $(20 + 32) - 46 = 6$.