2010 Q20

20. In a room, $\frac{2}{5}$ of all the people are wearing gloves, and $\frac{3}{4}$ of the people are wearing hats. What is the minimum number of people in the room wearing both a hat and gloves?

(A) 3 **(B)** 5 **(C)** 8 **(D)** 15 **(E)** 20

20. **Answer (A):** Because $\frac{2}{5}$ and $\frac{3}{4}$ of the people in the room are whole numbers, the number of people in the room is a multiple of both 5 and 4. The least common multiple of 4 and 5 is 20, so the minimum number of people in the room is 20. If $\frac{2}{5}$ of 20 people are wearing gloves, then 8 people are wearing gloves. If $\frac{3}{4}$ of 20 people are wearing hats, then 15 are wearing hats. The minimum number wearing gloves and hats occurs if the 5 not wearing hats are each wearing gloves. This leaves $8-5=3$ people wearing both gloves and hats.

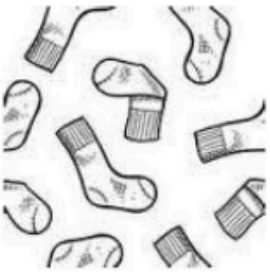
OR

If 8 are wearing gloves and 15 are wearing hats, then $8+15$ are wearing gloves and/or hats. There is a minimum of 20 people in the room, so $23-20=3$ people are wearing both a hat and gloves.

2015 Q20

20. Ralph went to the store and bought 12 pairs of socks for a total of \$24. Some of the socks he bought cost \$1 a pair, some of the socks he bought cost \$3 a pair, and some of the socks he bought cost \$4 a pair. If he bought at least one pair of each type, how many pairs of \$1 socks did Ralph buy?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8



20. **Answer (D):** If Ralph buys 6 pairs of \$1 socks, then the other 6 pairs of socks would cost at least \$19 making the total cost more than \$24. Buying fewer than 6 pairs of \$1 socks would make Ralph's cost even higher. If he bought 8 pairs of \$1 socks, then the other 4 pairs would cost less than \$16 making the total cost less than \$24. Buying more than 8 pairs of \$1 socks would make his total cost even lower. So Ralph bought 7 pairs of \$1 socks, 3 pairs of \$3 socks, and 2 pairs of \$4 socks.

OR

Let a, b and c be the number of pairs of \$1, \$3 and \$4 socks, respectively. Then $a+b+c=12$ and $a+3b+4c=24$. Subtracting the first equation from the second gives $2b+3c=12$. Since 3 is a factor of both 12 and $3c$, 3 must also be a factor of $2b$. Since $c>0$, it follows that $b=3$, $c=2$, and $a=7$.