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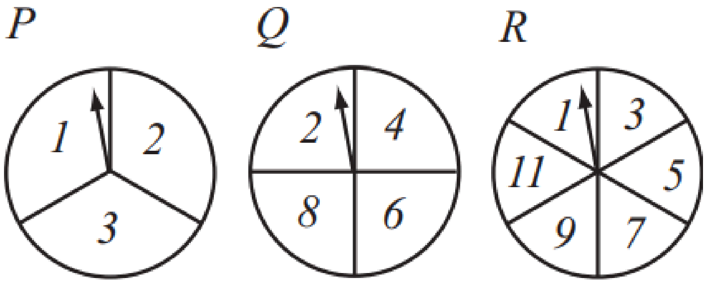
2006 Q17

17. Jeff rotates spinners P, Q and R and adds the resulting numbers. What is the probability that his sum is an odd number?



(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

17. **(B)** Because the sum of a number from spinner Q and a number from spinner R is always odd, the sum of the numbers on the three spinners will be odd exactly when the number from spinner P is even. Because 2 is the only even number on spinner P, the probability of getting an odd sum is $\frac{1}{3}$.

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2001 Q18

18. Two dice are thrown. What is the probability that the product of the two numbers is a multiple of 5?

(A) $\frac{1}{36}$ (B) $\frac{1}{18}$ (C) $\frac{1}{6}$ (D) $\frac{11}{36}$ (E) $\frac{1}{3}$

18. (D) There would be $6\times 6 = 36$ entries in the table if it were complete, but only the 11 entries that are multiples of 5 are shown. The probability of getting a multiple of 5 is $11/36$.

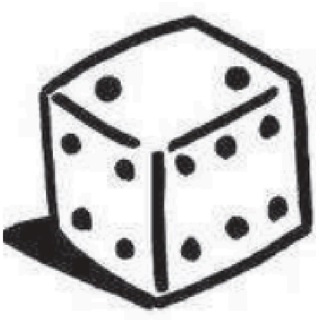
$\times$	1	2	3	4	5	6
1					5	
2					10	
3					15	
4					20	
5	5	10	15	20	25	30
6					30	

OR

Probability questions are sometimes answered by calculating the ways the event will NOT happen, then subtracting. In this problem the 1, 2, 3, 4 and 6 faces are paired to create $5 \times 5 = 25$ number pairs whose product is NOT multiples of 5. This leaves $36 - 25 = 11$ ways to get a multiple of 5, so the probability is $11/36$.

2011 Q18

18. A fair six-sided die is rolled twice. What is the probability that the first number that comes up is greater than or equal to the second number?



(A) $\frac{1}{6}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{7}{12}$ **(E)** $\frac{5}{6}$

18. **Answer (D):** Make a table of 36 possible equally-likely outcomes. The first number is greater than or equal to the second in the 21 cases indicated by the asterisks, so the probability is $\frac{21}{36} = \frac{7}{12}$.

	1	2	3	4	5	6
1	$1, 1*$	$1, 2$	$1, 3$	$1, 4$	$1, 5$	$1, 6$
2	$2, 1*$	$2, 2*$	$2, 3$	$2, 4$	$2, 5$	$2, 6$
3	$3, 1*$	$3, 2*$	$3, 3*$	$3, 4$	$3, 5$	$3, 6$
4	$4, 1*$	$4, 2*$	$4, 3*$	$4, 4*$	$4, 5$	$4, 6$
5	$5, 1*$	$5, 2*$	$5, 3*$	$5, 4*$	$5, 5*$	$5, 6$
6	$6, 1*$	$6, 2*$	$6, 3*$	$6, 4*$	$6, 5*$	$6, 6*$

OR

In 6 of the 36 possible outcomes the two numbers are equal. The first number is greater than the second in half of the remaining 30 outcomes, so the first number is greater than or equal to the second in $6 + 15 = 21$ outcomes. The probability is $\frac{21}{36} = \frac{7}{12}$.

2014 Q18

18. Four children were born at City Hospital yesterday. Assume each child is equally likely to be a boy or a girl. Which of the following outcomes is most likely?

(A) all 4 are boys **(B)** all 4 are girls **(C)** 2 are girls and 2 are boys

(D) 3 are of one gender and 1 is of the other gender

(E) all of these outcomes are equally likely

18. **Answer (D):** The 16 equally likely outcomes may be grouped as follows:

4 boys: BBBB

3 boys, 1 girl: BBBG, BBGB, BGBB, GBBB

2 boys, 2 girls: BBGG, BGBG, BGGB, GBBG, GBGB, GGBB

1 boy, 3 girls: BGGG, GBGG, GGBG, GGGB

4 girls: GGGG

There are 8 equally likely outcomes that produce 3 of one gender and 1 of the other gender, so that result is most likely.

1998 Q19

19. Tamika selects two different numbers at random from the set $\{8, 9, 10\}$ and adds them. Carlos takes two different numbers at random from the set $\{3, 5, 6\}$ and multiplies them. What is the probability that Tamika's result is greater than Carlos' result?

(A) $\frac{4}{9}$ **(B)** $\frac{5}{9}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{2}{3}$

19. **Answer (A):** Tamika can get the numbers $8 + 9 = 17$, $8 + 10 = 18$, or $9 + 10 = 19$. Carlos can get $3 \times 5 = 15$, $3 \times 6 = 18$, or $5 \times 6 = 30$. The possible ways to pair these are: $(17, 15)$, $(17, 18)$, $(17, 30)$, $(18, 15)$, $(18, 18)$, $(18, 30)$, $(19, 15)$, $(19, 18)$, $(19, 30)$. Four of these nine pairs show Tamika with a higher result, so the probability is $4/9$.

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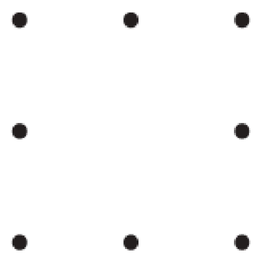
2008 Q19

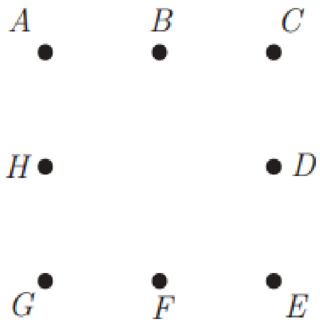
19. Eight points are spaced at intervals of one unit around a 2×2 square, as shown. Two of the 8 points are chosen at random. What is the probability that the points are one unit apart?



(A) $\frac{1}{4}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{4}{11}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{4}{7}$

19. **Answer (B):** Choose two points. Any of the 8 points can be the first choice, and any of the 7 other points can be the second choice. So there are $8 \times 7 = 56$

ways of choosing the points in order. But each pair of points is counted twice, so there are $\frac{56}{2} = 28$ possible pairs.



Label the eight points as shown. Only segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{FG}$, $\overline{GH}$ and $\overline{HA}$ are 1 unit long. So 8 of the 28 possible segments are 1 unit long, and the probability that the points are one unit apart is $\frac{8}{28} = \frac{2}{7}$.

OR

Pick the two points, one at a time. No matter how the first point is chosen, exactly 2 of the remaining 7 points are 1 unit from this point. So the probability of the second point being 1 unit from the first is $\frac{2}{7}$.

1995 Q20

20. **Diana and Apollo each roll a standard die obtaining a number at random from 1 to 6. What is the probability that Diana's number is larger than Apollo's number?**

(A) $\frac{1}{3}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{17}{36}$ **(E)** $\frac{1}{2}$

20. **(B)** There are $6 \times 6 = 36$ possible outcomes of rolling the dice. Since Diana and Apollo roll the same number in 6 of these, there are 30 in which the numbers on the two dice are different. By symmetry, Diana's number is larger than Apollo's number in exactly half of these. Thus the requested probability is $\frac{15}{36} = \frac{5}{12}$.

OR

Let (d, a) represent "*Diana rolled d and Apollo rolled a.*" List the 36 outcomes and mark those where $d > a$.

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	**(3,2)**	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	**(4,2)**	**(4,3)**	(4,4)	(4,5)	(4,6)
(5,1)	**(5,2)**	**(5,3)**	**(5,4)**	(5,5)	(5,6)
(6,1)	**(6,2)**	**(6,3)**	**(6,4)**	**(6,5)**	(6,6)

Since there are 15 marked pairs, the probability that Diana rolls a larger number than Apollo is $15/36 = 5/12$.

1997 Q20

20. A pair of 8-sided dice have sides numbered 1 through 8. Each side has the same probability (chance) of landing face up. The probability that the product of the two numbers on the sides that land face-up exceeds 36 is

(A) $\frac{5}{32}$ **(B)** $\frac{11}{64}$ **(C)** $\frac{3}{16}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{1}{2}$

20. **(A)** There are 64 equally likely possibilities for the numbers on the two dice. Of these, only (5, 8), (6, 7), (6, 8), (7, 6), (7, 7), (7, 8), (8, 5), (8, 6), (8, 7), and (8, 8) give products exceeding 36, so the probability of this occurring is $\frac{10}{64} = \frac{5}{32}$.

OR

Make a table for the sample space:

x	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32
5	5	10	15	20	25	30	35	(40)
6	6	12	18	24	30	36	(42)	(48)
7	7	14	21	28	35	(42)	(49)	(56)
8	8	16	24	32	(40)	(48)	(56)	(64)

The ten circled products exceed 36, so the probability of this occurring is $\frac{10}{64}$ or $\frac{5}{32}$.

2017 Q20

20. An integer between 1000 and 9999, inclusive, is chosen at random. What is the probability that it is an odd integer whose digits are all distinct?

(A) $\frac{14}{75}$ **(B)** $\frac{56}{225}$ **(C)** $\frac{107}{400}$ **(D)** $\frac{7}{25}$ **(E)** $\frac{9}{25}$

20. **Answer (B):** There are 9000 integers between 1000 and 9999 inclusive. For an integer to be odd it must end in 1, 3, 5, 7, or 9. So there are 5 choices for the units digit. For a number to be between 1000 and 9999 the thousands digit must be nonzero and so there are now 8 choices for the thousands digit. For the hundreds digit there are 8 choices and for the tens digit there are 7 choices for a total number of $5 \cdot 8 \cdot 8 \cdot 7 = 2240$ choices. So the probability is $\frac{2240}{9000} = \frac{56}{225}$.

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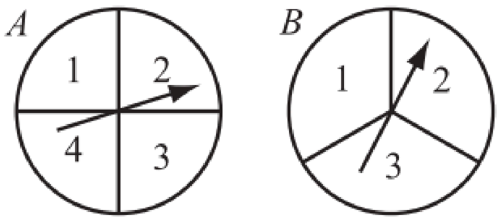
2004 Q21

21. Spinners A and B are spun. On each spinner, the arrow is equally likely to land on each number. What is the probability that the product of the two spinners' numbers is even?



(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

21. **(D)** In eight of the twelve outcomes the product is even: 1×2, 2×1, 2×2, 2×3, 3×2, 4×1, 4×2, 4×3. In four of the twelve, the product is odd: 1×1, 1×3, 3×1, 3×3. So the probability that the product is even is $\frac{8}{12}$ or $\frac{2}{3}$.

OR

To get an odd product, the result of both spins must be odd. The probability of odd is $\frac{1}{2}$ on Spinner A and $\frac{2}{3}$ on Spinner B. So the probability of an odd product is $\left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = \frac{1}{3}$. The probability of an even product, then, is $1 - \frac{1}{3} = \frac{2}{3}$.