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1994 Q21

21. A gumball machine contains 9 red, 7 white, and 8 blue gumballs. The least number of gumballs a person must buy to be *sure* of getting four gumballs of the same color is

(A) 8 (B) 9 (C) 10 (D) 12 (E) 18

21. (C) It is *possible* to get four gumballs of the same color by buying 4, 5, 6, 7, 8, or 9. However, the first nine gumballs *might* consist of three of each color, so nine gumballs will *not guarantee* four of the same color. In this case, the tenth gumball must match one of the previous colors, giving four of that color.

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1996 Q21

21. How many subsets containing three different numbers can be selected from the set

$$\{89, 95, 99, 132, 166, 173\}$$

so that the sum of the three numbers is even?

(A) 6 (B) 8 (C) 10 (D) 12 (E) 24

21. **(D)** The sum of three numbers is even if all three numbers are even, or if two numbers are odd and one is even. Since there are only two even numbers in the set, it follows that the three numbers must include two odd numbers and one even. The possibilities are:

$$\{89, 95, 132\} \quad \{89, 99, 132\} \quad \{89, 173, 132\}$$
$$\{89, 95, 166\} \quad \{89, 99, 166\} \quad \{89, 173, 166\}$$

$$\{95, 99, 132\} \quad \{95, 173, 132\} \quad \{99, 173, 132\}$$
$$\{95, 99, 166\} \quad \{95, 173, 166\} \quad \{99, 173, 166\}.$$

Thus there are 12 possibilities.

OR

Let O stand for odd and E stand for even. The numbers given are O, O, O, E, E and O. There are only two ways that the sum of three numbers is even.

Case I: $E + E + E = E$, and

Case II: $O + O + E = E$.

Since there are only two even numbers, **Case I** cannot happen. For **Case II**, $O + O + E$, counting choices yields 4 choices for the first odd, 3 remaining choices for the second odd, and 2 choices for the even, for a total of $4 \times 3 \times 2 = 24$ choices. However, since $O_1 + O_2 + E = O_2 + O_1 + E$, the number of choices is reduced by a factor of 2. Hence there are $24/2 = 12$ choices.

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2013 Q21

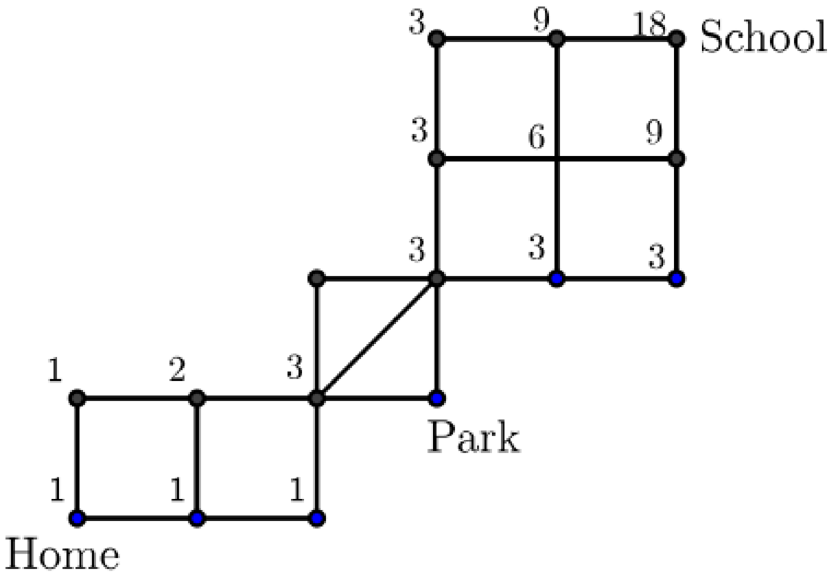
21. Samantha lives 2 blocks west and 1 block south of the southwest corner of City Park. Her school is 2 blocks east and 2 blocks north of the northeast corner of City Park. On school days she bikes on streets to the southwest corner of City Park, then takes a diagonal path through the park to the northeast corner of City Park, and then bikes on streets to school. If her route is as short as possible, how many different routes can she take?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 18

21. **Answer (E):** There are 3 ways she can bike from home to the southwest corner of the park, EEN, ENE, or NEE. There are 6 ways to bike from the northeast corner of the park to school, EENN, ENEN, ENNE, NEEN, NENE, or NNEE. So there are $6 \cdot 3 = 18$ routes.

OR

Using a Pascal's Triangle approach starting from the house to the school, count the routes to each intermediate point with the following diagram, moving only north or east at each corner.



1985 Q22

22. Assume every 7-digit whole number is a possible telephone number except those which begin with 0 or 1. What fraction of telephone numbers begin with 9 and end with 0?

A) $\frac{1}{63}$ B) $\frac{1}{80}$ C) $\frac{1}{81}$ D) $\frac{1}{90}$ E) $\frac{1}{100}$

22. (B) There are 10 digits. Excluding 0 and 1 leaves 8 digits. Thus $\frac{1}{8}$ of all telephone numbers begin with 9. Of these, $\frac{1}{10}$ end with 0 giving $\frac{1}{8} \times \frac{1}{10} = \frac{1}{80}$ which begin with 9 and end with 0.

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1989 Q22

22. The letters A,J,H,S,M,E and the digits 1,9,8,9 are "cycled" separately as follows and put in a numbered list:

AJHSME 1989

1. JHSMEA 9891
2. HSMEAJ 8919
3. SMEAJH 9198

....

What is the number of the line on which AJHSME 1989 will appear for the first time?

A) 6 B) 10 C) 12 D) 18 E) 24

22. C The 6 letters will go through a complete cycle every 6 lines and the 4 numbers every 4 lines. The letters and numbers together will go through a complete cycle every LCM (6,4) = 12 times. Thus AJHSME 1989 occurs for the first time on the 12th line.

1993 Q22

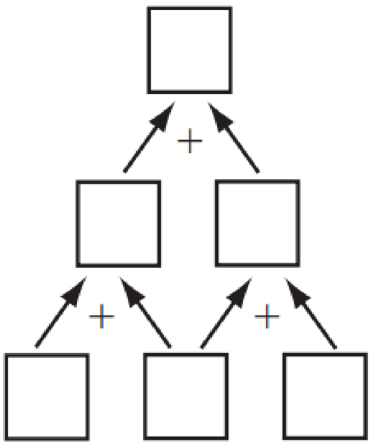
22. Pat Peano has plenty of 0's, 1's, 3's, 4's, 5's, 6's, 7's, 8's and 9's, but he has only twenty-two 2's. How far can he number the pages of his scrapbook with these digits?

(A) 22 **(B)** 99 **(C)** 112 **(D)** 119 **(E)** 199

22. **(D)** Ten 2's are needed in the unit's place in counting to 100 and ten more 2's are used in the ten's place. With the remaining two 2's he can number 102 and 112 and continue all the way to 119 before needing another 2.

2006 Q22

22. Three different one-digit positive integers are placed in the bottom row of cells. Numbers in adjacent cells are added and the sum is placed in the cell above them. In the second row, continue the same process to obtain a number in the top cell. What is the difference between the largest and smallest numbers possible in the top cell?



(A) 16 **(B)** 24 **(C)** 25 **(D)** 26 **(E)** 35

22. **(D)** If the lower cells contain A, B and C, then the second row will contain $A + B$ and $B + C$, and the top cell will contain $A + 2B + C$. To obtain the smallest sum, place 1 in the center cell and 2 and 3 in the outer ones. The top number will be 7. For the largest sum, place 9 in the center cell and 7 and 8 in the outer ones. This top number will be 33. The difference is $33 - 7 = 26$.

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2009 Q22

22. How many whole numbers between 1 and 1000 do **not** contain the digit 1?

(A) 512 **(B)** 648 **(C)** 720 **(D)** 728 **(E)** 800

22. **Answer (D):** There are 8 one-digit positive integers, excluding 1. There are $8 \cdot 9 = 72$ two-digit integers that do not contain the digit 1. There are $8 \cdot 9 \cdot 9 = 648$ three-digit integers that do not contain the digit 1. There are $8 + 72 + 648 = 728$ integers between 1 and 1000 that do not contain the digit 1.

OR

Think of each number between 1 and 1000 as a three-digit number. For example, think of 2 as 002 and 27 as 027. There are $9^3 = 729$ three-digit numbers that do not use the digit 1. Because 000 does not represent a whole number between 1 and 1000, the total is 728.