2003 Q14

14. In this addition problem, each letter stands for a different digit.

$$\begin{array}{cccc} & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$

If $T = 7$ and the letter O represents an even number, what is the only possible value for W?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

14. **(D)** As given, $T = 7$. This implies that $F = 1$ and that O equals either 4 or 5. Since O is even, $O = 4$. Therefore, $R = 8$. Replacing letters with numerals gives

$$\begin{array}{cccc} & 7 & W & 4 \\ + & 7 & W & 4 \\ \hline 1 & 4 & U & 8 \end{array}$$

$W + W$ must be less than 10; otherwise, a 1 would be carried to the next column, and O would be 5. So $W < 5$. $W \neq 0$ because $W \neq U$, $W \neq 1$ because $F = 1$, $W \neq 2$ because if $W = 2$ then $U = 4 = O$, and $W \neq 4$ because $O = 4$. So $W = 3$.

The addition problem is

$$\begin{array}{cccc} & 7 & 3 & 4 \\ + & 7 & 3 & 4 \\ \hline 1 & 4 & 6 & 8 \end{array}$$

2008 Q14

14. Three As, three Bs and three Cs are placed in the nine spaces so that each row and column contain one of each letter. If A is placed in the upper left corner, how many arrangements are possible?

A		

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

14. **Answer (C):** There are only two possible spaces for the B in row 1 and only two possible spaces for the A in row 2. Once these are placed, the entries in the remaining spaces are determined.

The four arrangements are:

A	B	C
B	C	A
C	A	B

A	B	C
C	A	B
B	C	A

A	C	B
C	B	A
B	A	C

A	C	B
B	A	C
C	B	A

OR

The As can be placed either

A		
	A	
		A

or

A		
		A
	A	

In each case, the letter next to the top A can be B or C. At that point the rest of the grid is completely determined. So there are $2+2=4$ possible arrangements.

2001 Q14

14. Tyler has entered a buffet line in which he chooses one kind of meat, two different vegetables and one dessert. If the order of food items is not important, how many different meals might he choose?

Meat: beef, chicken, pork
Vegetables: baked beans, corn, potatoes, tomatoes
Dessert: brownies, chocolate cake, chocolate pudding, ice cream

(A) 4 (B) 24 (C) 72 (D) 80 (E) 144

14. (C) There are 3 choices for the meat and 4 for dessert.

There are 6 ways to choose the two vegetables. The first vegetable may be chosen in 4 ways and the second in 3 ways. This would seem to make 12 ways, but since the order is not important the 12 must be divided by 2. Otherwise, for example, both tomatoes/corn and corn/tomatoes would be included. The 6 choices are beans/corn, beans/potatoes, beans/tomatoes, corn/potatoes, corn/tomatoes and potatoes/tomatoes.

The answer is $3(4)(6)=72$.

2005 Q14

14. The Little Twelve Basketball Conference has two divisions, with six teams in each division. Each team plays each of the other teams in its own division twice and every team in the other division once. How many conference games are scheduled?

(A) 80 **(B)** 96 **(C)** 100 **(D)** 108 **(E)** 192

14. **(B)** Each team plays 10 games in its own division and 6 games against teams in the other division. So each of the 12 teams plays 16 conference games. Because each game involves two teams, there are $\frac{12\times16}{2} = 96$ games scheduled.

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1985 Q15

15. How many whole numbers between 100 and 400 contain the digit 2 ?

A) 100 B) 120 C) 138 D) 140 E) 148

15. (C) In addition to the 100 numbers from 200-299, there are 20 numbers ending in 2 (e.g., 112, 342) and 20 numbers with a ten's digit of 2 (e.g., 127, 325). But the numbers 122 and 322 are counted twice in this process, so there are a total of 100 + 20 + 20 - 2 = 138.

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1999 Q15

15. Bicycle license plates in Flatville each contain three letters. The first is chosen from the set {C,H,L,P,R}, the second from {A,I,O}, and the third from {D,M,N,T}.

When Flatville needed more license plates, they added two new letters. The new letters may both be added to one set or one letter may be added to one set and one to another set. What is the largest possible number of ADDITIONAL license plates than can be made by adding two letters?

(A) 24 **(B)** 30 **(C)** 36 **(D)** 40 **(E)** 60

15. **Answer (D):** Before new letters were added, five different letters could have been chosen for the first position, three for the second, and four for the third. This means that $5 \cdot 3 \cdot 4 = 60$ plates could have been made.
If two letters are added to the second set, then $5 \cdot 5 \cdot 4 = 100$ plates can be made. If one letter is added to each of the second and third sets, then $5 \cdot 4 \cdot 5 = 100$ plates can be made. None of the other four ways to place the two letters will create as many plates. So, $100 - 60 = 40$ ADDITIONAL plates can be made.

Note: Optimum results can usually be obtained in such problems by making the factors as nearly equal as possible.