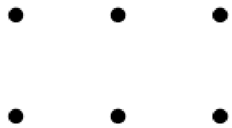
1 / 15

2005 Q21

21. How many distinct triangles can be drawn using three of the dots below as vertices?



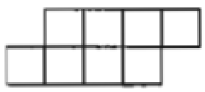
(A) 9 **(B)** 12 **(C)** 18 **(D)** 20 **(E)** 24

21. **(C)** To make a triangle, select as vertices two dots from one row and one from the other row. To select two dots in the top row, decide which dot is not used. This can be done in three ways. There are also three ways to choose one dot to use from the bottom row. So there are $3 \times 3 = 9$ triangles with two vertices in the top row and one in the bottom. Similarly, there are nine triangles with one vertex in the top row and two in the bottom. This gives a total of $9 + 9 = 18$ triangles.
Note: Can you find the four noncongruent triangles?

2 / 15

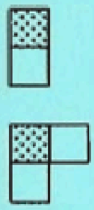
1992 Q22

22. Eight 1×1 square tiles are arranged as shown so their outside edges form a polygon with a perimeter of 14 units. Two additional tiles of the same size are added to the figure so that at least one side of each tile is shared with a side of one of the squares in the original figure. Which of the following could be the perimeter of the new figure?

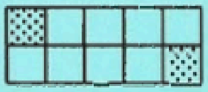


(A) 15 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

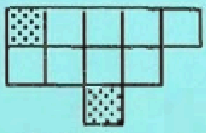
22. **(C)** When a new tile is added to the original figure, it may have one or two sides in common with the given tiles, as shown. When a tile shares one side, the original perimeter is increased by 2. When a tile shares two sides, there is no change in the perimeter. By adding two tiles, the only possible changes to the perimeter are increases of 0, 2 or 4. Hence, the possible values of the perimeter are 14, 16 or 18.



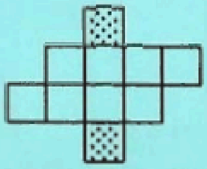
Note. Examples of the three possibilities are shown.



original = 14
new perimeter = 14



original = 14
new perimeter = 16



original = 14
new perimeter = 18

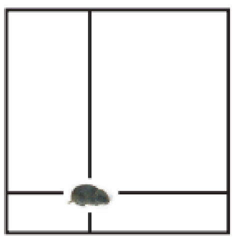
3 / 15

2007 Q22

22. A lemming sits at a corner of a square with side length 10 meters. The lemming runs 6.2 meters along a diagonal toward the opposite corner. It stops, makes a 90° right turn and runs 2 more meters. A scientist measures the shortest distance between the lemming and each side of the square. What is the average of these four distances in meters?

(A) 2 **(B)** 4.5 **(C)** 5 **(D)** 6.2 **(E)** 7

22. **(C)** Wherever the lemming is inside the square, the sum of the distances to the two horizontal sides is 10 meters and the sum of the distances to the two vertical sides is 10 meters. Therefore the sum of all four distances is 20 meters, and the average of the four distances is $\frac{20}{4} = 5$ meters.



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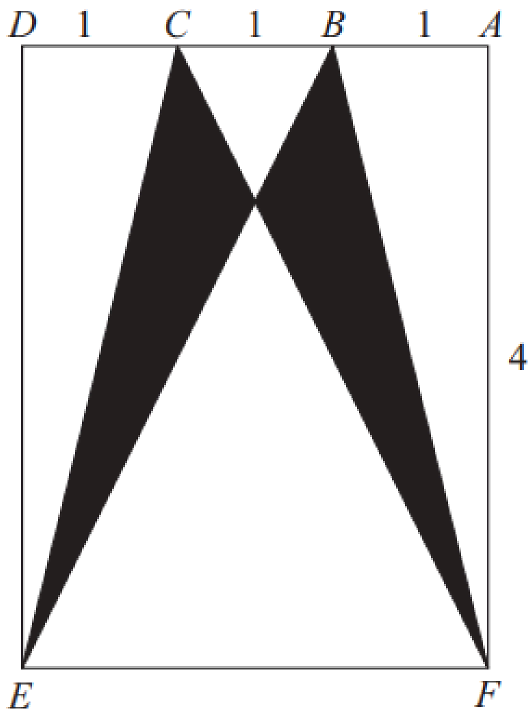
2016 Q22

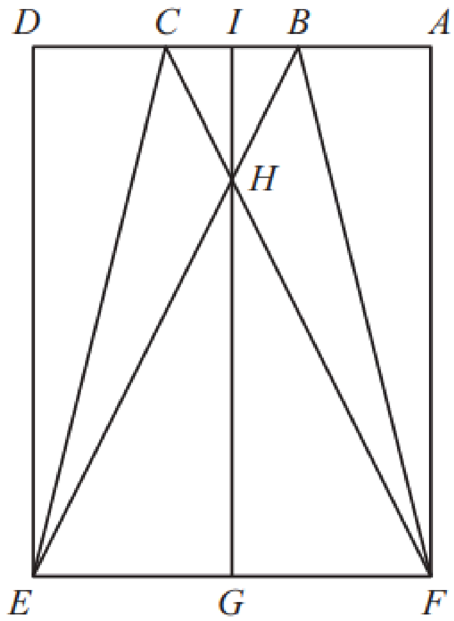
22. Rectangle $DEFA$ below is a 3×4 rectangle with $DC = CB = BA = 1$. The area of the "bat wings" (the shaded area) is

(A) 2 **(B)** $2\frac{1}{2}$ **(C)** 3 **(D)** $3\frac{1}{2}$ **(E)** 4



22. Answer (C):

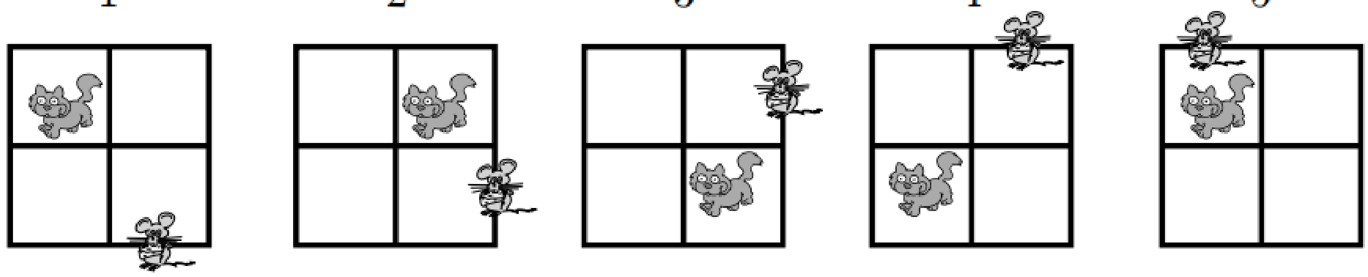
The area of $\triangle BCE$ is $\frac{1}{2}(1)(4) = 2$. Triangles $\triangle CBH$ and $\triangle EFH$ are similar. Since $CB = \frac{1}{3}EF$, it follows that $IH = \frac{1}{3}GH = \frac{1}{4}IG = 1$. The area of $\triangle CBH$ is $\frac{1}{2}$, so the area of $\triangle ECH$ is $2 - \frac{1}{2} = \frac{3}{2}$. Thus the batwing's area is 3.



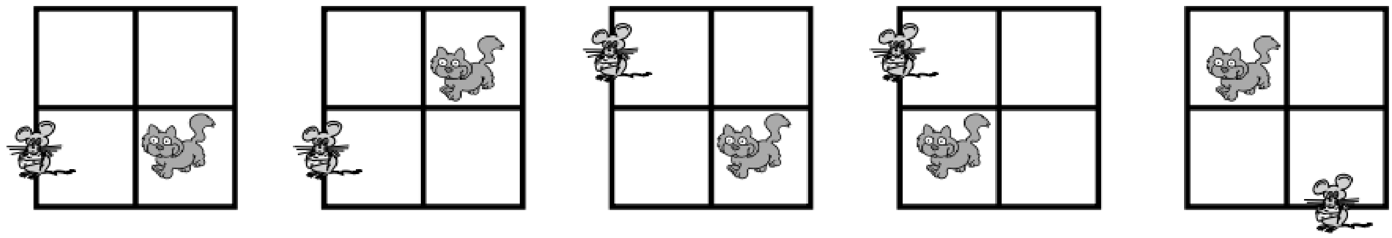
2003 Q23

23. In the pattern below, the cat moves clockwise through the four squares and the mouse moves counterclockwise through the eight exterior segments of the four squares.

1 2 3 4 5

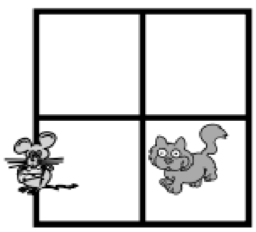


If the pattern is continued, where would the cat and mouse be after the 247th move?



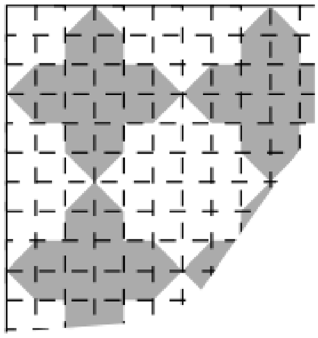
(A) (B) (C) (D) (E)

23. **(A)** There are four different positions for the cat in the 2×2 array, so after every fourth move, the cat will be in the same location. Because $247 = 4 \times 61 + 3$, the cat will be in the 3rd position clockwise from the first, or the lower right quadrant. There are eight possible positions for the mouse. Because $247 = 8 \times 30 + 7$, the mouse will be in the 7th position counterclockwise from the first, or the left-hand side of the lower left quadrant.



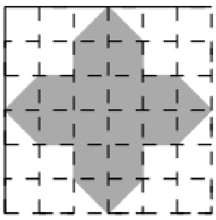
2002 Q23

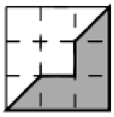
23. A corner of a tiled floor is shown. If the entire floor is tiled in this way and each of the four corners looks like this one, then what fraction of the tiled floor is made of darker tiles?



(A) $\frac{1}{3}$ **(B)** $\frac{4}{9}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{5}{9}$ **(E)** $\frac{5}{8}$

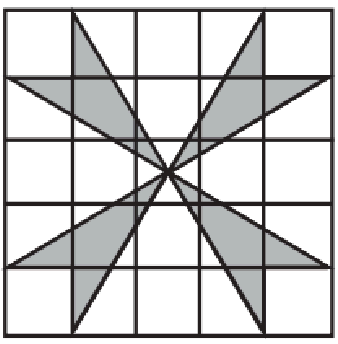
23. **(B)** The 6×6 square in the upper left-hand region is tessellated, so finding the proportion of darker tiles in this region will answer the question. The top left-hand corner of this region is a 3×3 square that has $3 + 2\left(\frac{1}{2}\right) = 4$ darker tiles. So $\frac{4}{9}$ of the total area will be made of darker tiles.





2007 Q23

23. What is the area of the shaded pinwheel shown in the 5×5 grid?



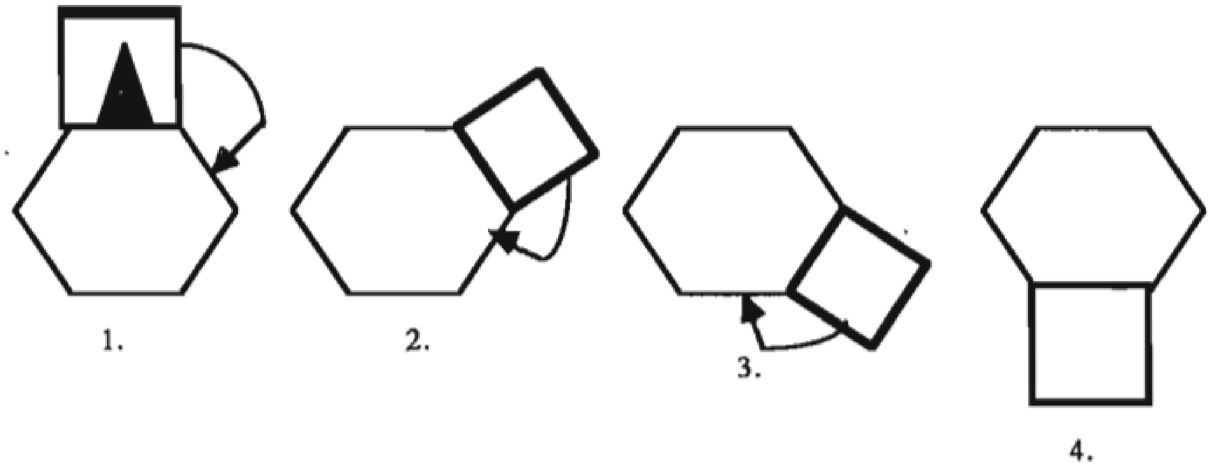
(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

23. **(B)** Find the area of the unshaded portion of the 5×5 grid, then subtract the unshaded area from the total area of the grid. The unshaded triangle in the middle of the top of the 5×5 grid has a base of 3 and an altitude of $\frac{5}{2}$. The four unshaded triangles have a total area of $4 \times \frac{1}{2} \times 3 \times \frac{5}{2} = 15$ square units. The four corner squares are also unshaded, so the shaded pinwheel has an area of $25 - 15 - 4 = 6$ square units.

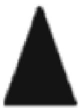
8 / 15

1988 Q24

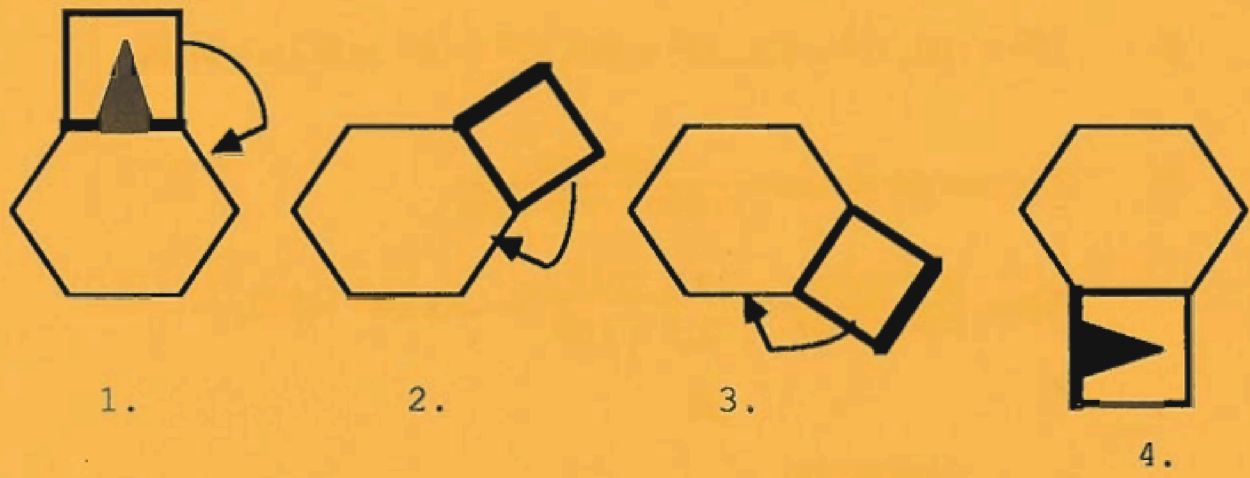
24.



The square in the first diagram "rolls" clockwise around the fixed regular hexagon until it reaches the bottom. In which position will the solid triangle be in diagram 4?

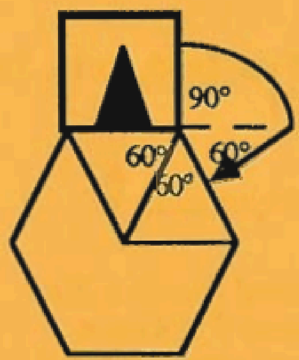
A)  B)  C)  D)  E) 

24. A



Keep track of the "bottom" side of the square in the first figure. In the fourth figure, it will appear on the left, so the solid triangle will be in the position shown in (A).

OR



Each time the square "rolls" to the next edge of the hexagon, it turns through an angle of 150°. In going from the top to the bottom of the hexagon, the square makes three such turns for a total of 3(150) = 450°. This 450° represents one complete revolution and 1/4 of a second revolution.

1993 Q24

24. What number is directly above 142 in this array of numbers?

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 2 & 3 & 4 & & \\ & 5 & 6 & 7 & 8 & 9 & \\ 10 & 11 & 12 & \cdots & & & \end{array}$$

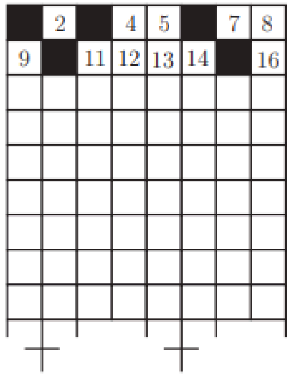
(A) 99 **(B)** 119 **(C)** 120 **(D)** 121 **(E)** 122

24. **(C)** After completing some more rows, the pattern of squares as the last entry in each row becomes apparent. Thus, the line containing 142 ends in 144, and the line above it ends in 121. Therefore 121 is directly above 143, and 120 is above 142.

$$\begin{array}{ccccccccc} & & & \boxed{1} & & & & & \\ & & 2 & 3 & \boxed{4} & & & & \\ & 5 & 6 & 7 & 8 & \boxed{9} & & & \\ 10 & 11 & 12 & 13 & 14 & 15 & \boxed{16} & & \\ & & & \vdots & & & & & \\ & & & & \cdots & 119 & 120 & \boxed{121} & \\ & & & & \cdots & 141 & 142 & 143 & \boxed{144} \end{array}$$

1998 Q24

24. A rectangular board of 8 columns has squares numbered beginning in the upper left corner and moving left to right so row one is numbered 1 through 8, row two is 9 through 16, and so on. A student shades square 1, then skips one square and shades square 3, skip two squares and shades square 6, ships 3 squares and shades square 10, and continues in this way until there is at least one shaded square in each column. What is the number of the shaded square that first achieves this result?

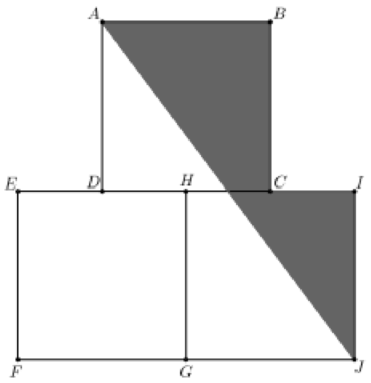


(A) 36 **(B)** 64 **(C)** 78 **(D)** 91 **(E)** 120

24. **Answer (E):** The numbers in the first column all have remainders of 1 when divided by 8, those of the second column have remainders of 2 when divided by 8, and so on. We need to find numbered squares so that each remainder 0 through 7 appears at least once. The squares that are shaded are numbered 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, and the remainders upon dividing by 8 are 1, 3, 6, 2, 7, 5, 4, 4, 5, 7, 2, 6, 3, 1, 0. Thus, we must shaded square 120 to obtain the first shaded square in the last column.

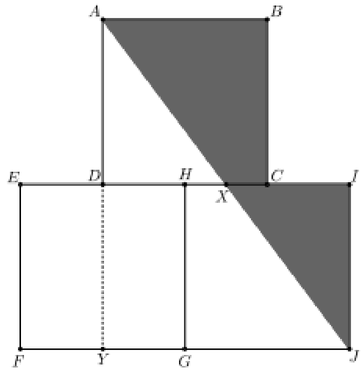
2013 Q24

24. Squares $ABCD$, $EFGH$, and $GHIJ$ are equal in area. Points C and D are the midpoints of sides IH and HE, respectively. What is the ratio of the area of the shaded pentagon $AJICB$ to the sum of the areas of the three squares?



(A) $\frac{1}{4}$ **(B)** $\frac{7}{24}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{5}{12}$

24. **Answer (C):**



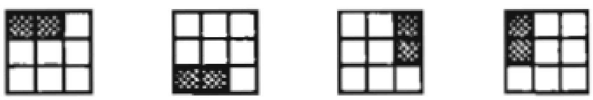
Let the length of the side of each square be 1 and extend side AD to Y as shown. The total area of the three squares is 3. The unshaded area is area $(EDYF)$ + area $(AYJ) = 1(\frac{1}{2}) + \frac{1}{2} \cdot \frac{3}{2} \cdot 2 = \frac{1}{2} + \frac{3}{2} = 2$, so the shaded area is 1 and the desired ratio is $\frac{1}{3}$.

OR

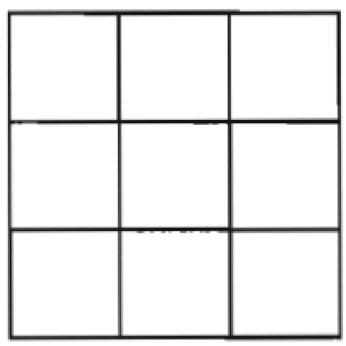
Label point X as shown. Intuitively, rotating $\triangle XIJ$ $180°$ about X takes it to $\triangle XDA$ so the shaded area is the same as the area of square $ABCD$ and the desired ratio is $\frac{1}{3}$. More precisely, segments AX, XD, and DA are parallel to segments JX, XI, and IJ, respectively. Also, $DA = IJ$, so $\triangle ADX$ is congruent to $\triangle JIX$.

1990 Q25

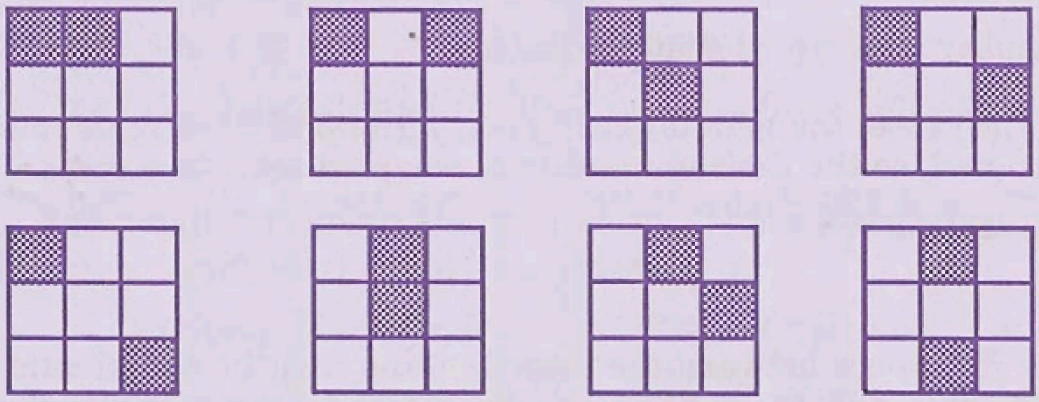
25. How many different patterns can be made by shading exactly two of the nine squares? Patterns that can be matched by flips and/or turns are not considered different. For example, the patterns shown below are **not** considered different.



A) 3 B) 6 C) 8 D) 12 E) 18



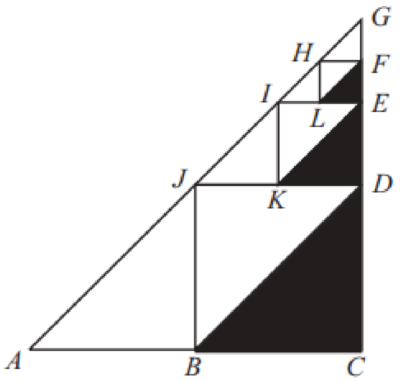
25. C There are 8. Be systematic. You can begin with the five cases that have one corner square. Then consider the other three cases that do not have a corner square.



1999 Q25

25. Points B, D, and J are midpoints of the sides of right triangle ACG. Points K, E, I are midpoints of the sides of triangle JDG, etc. If the dividing and shading process is done 100 times(the first three are shown) and $AC = CG = 6$, then the total area of the shaded triangles is nearest

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10



25. **Answer (A):** At each stage the area of the shaded triangle is one-third of the trapezoidal region not containing the smaller triangle being divided in the next step. Thus, the total area of the shaded triangles comes closer and closer to one-third of the area of the triangular region ACG and this is $\frac{1}{3} \cdot \frac{1}{2} \cdot 6 \cdot 6 = 6$. The shaded areas for the first six stages are: 4.5, 5.625, 5.906, 5.976, 5.994, and 5.998.

These are the calculations for the first three steps.

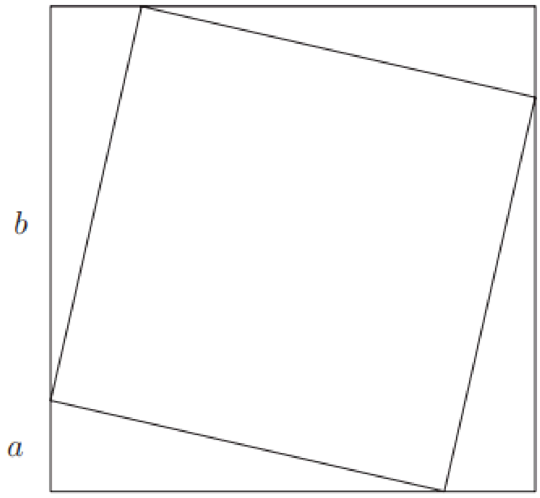
$\frac{1}{2} \cdot \frac{6}{2} \cdot \frac{6}{2} = 4.5$

$\frac{1}{2} \cdot \frac{6}{2} \cdot \frac{6}{2} + \frac{1}{2} \cdot \frac{6}{4} \cdot \frac{6}{4} = 4.5 + 1.125 = 5.625$

$\frac{1}{2} \cdot \frac{6}{2} \cdot \frac{6}{2} + \frac{1}{2} \cdot \frac{6}{4} \cdot \frac{6}{4} + \frac{1}{2} \cdot \frac{6}{8} \cdot \frac{6}{8} = 5.625 + 0.281 = 5.906$

2012 Q25

25. A square with area 4 is inscribed in a square with area 5, with one vertex of the smaller square on each side of the larger square. A vertex of the smaller square divides a side of the larger square into two segments, one of length a and the other of length b. What is the value of ab?



(A) $\frac{1}{5}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{2}$ **(D)** 1 **(E)** 4

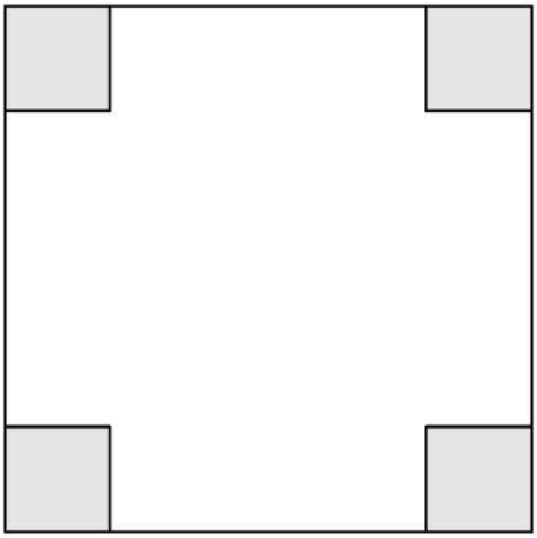
25. **Answer (C):** The area of the region inside the larger square and outside the smaller square has total area $5-4=1$ and is equal to the area of four congruent right triangles, each with one side of length a and the other of length b. The area of each triangle is $\frac{1}{4}$. If $\frac{1}{2}ab=\frac{1}{4}$, then $ab=\frac{1}{2}$.

15 / 15

2015 Q25

25. One-inch squares are cut from the corners of this 5 inch square. What is the area in square inches of the largest square that can be fitted into the remaining space?

(A) 9 **(B)** $12\frac{1}{2}$ **(C)** 15 **(D)** $15\frac{1}{2}$ **(E)** 17



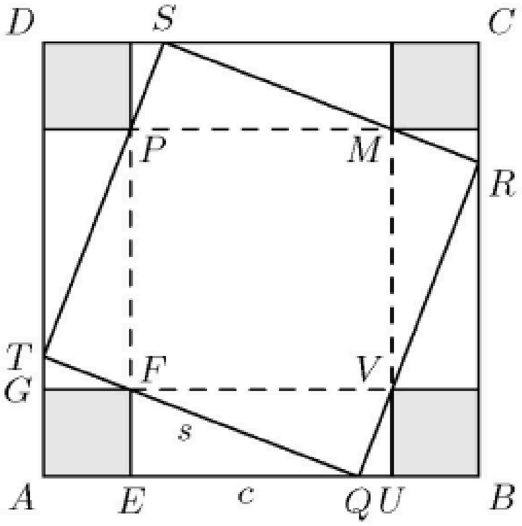
25. **Answer (C):** Let $EQ = c$ and $TQ = s$ as indicated in the figure. Triangles QUV and FEQ are similar since $\angle FQE$ and $\angle QVU$ are congruent because both are complementary to $\angle VQU$. So

$$\frac{QU}{UV} = \frac{FE}{EQ}$$

and thus $QU = \frac{1}{c}$. Then $AB = 1 + c + 1/c + 1 = 5$ and so $c + 1/c = 3$. Since the area of square $ABCD$ equals the sum of areas of square $QRST$, four unit squares, four $1 \times c$ triangles, and four $\frac{1}{c} \times 1$ triangles, it follows that

$$\begin{aligned} 25 &= s^2 + 4\left(1 + \frac{c}{2} + \frac{1}{2c}\right) \\ &= s^2 + 4 + 2\left(c + \frac{1}{c}\right) \\ &= s^2 + 4 + 2 \cdot 3 \end{aligned}$$

Therefore, the area of square $QRST = s^2 = 15$.



OR

Square $FVMP$ has area 9, the four triangles FQV, VRM, MSP, and PTF each have area $\frac{3}{2}$. So the area of square $STQR$ is $9 + 6 = 15$.