2017 Q15

15. In the arrangement of letters and numerals below, by how many different paths can one spell $AMC8$? Beginning at the A in the middle, a path allows only moves from one letter to an adjacent (above, below, left, or right, but not diagonal) letter. One example of such a path is traced in the picture.

	8	C	8	
8	C	M	C	8
C	M	A	M	C
8	C	M	C	8
	8	C	8	

(A) 8 **(B)** 9 **(C)** 12 **(D)** 24 **(E)** 36

15. **Answer (D):** Starting at A, there are 4 choices for the M, each of which is followed by 3 choices for the C, each of which is followed by 2 choices for the 8. So all together, there are $4 \cdot 3 \cdot 2 = 24$ paths.

1991 Q16

16. The 16 squares on a piece of paper are numbered as shown in the diagram. While lying on a table, the paper is folded in half four times in the following sequence:
(1) fold the top half over the bottom half
(2) fold the bottom half over the top half
(3) fold the right half over the left half
(4) fold the left half over the right half.
Which numbered square is on top after step 4?

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

(A) 1 **(B)** 9 **(C)** 10 **(D)** 14 **(E)** 16

16. The 16 squares on a piece of paper are numbered as shown in the diagram. While lying on a table, the paper is folded in half four times in the following sequence:
(1) fold the top half over the bottom half
(2) fold the bottom half over the top half
(3) fold the right half over the left half
(4) fold the left half over the right half.
Which numbered square is on top after step 4?

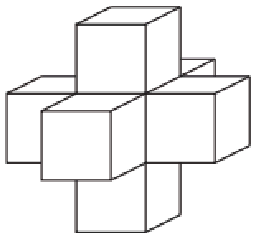
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

(A) 1 **(B)** 9 **(C)** 10 **(D)** 14 **(E)** 16

16. **(B)** A fold from the top leaves #9-16 on the bottom.
A fold from the bottom leaves #9-12 on the bottom.
A fold from the right leaves #9 and #10 on the bottom.
A fold from the left leaves #10 on the bottom with number #9 moving to the top.

2008 Q16

16. A shape is created by joining seven unit cubes, as shown. What is the ratio of the volume in cubic units to the surface area in square units?



(A) $1 : 6$ **(B)** $7 : 36$ **(C)** $1 : 5$ **(D)** $7 : 30$ **(E)** $6 : 25$

16. **Answer (D):** The volume is $7 \times 1 = 7$ cubic units. Six of the cubes have 5 square faces exposed. The middle cube has no face exposed. So the total surface area of the figure is $5 \times 6 = 30$ square units. The ratio of the volume to the surface area is $7 : 30$.

OR

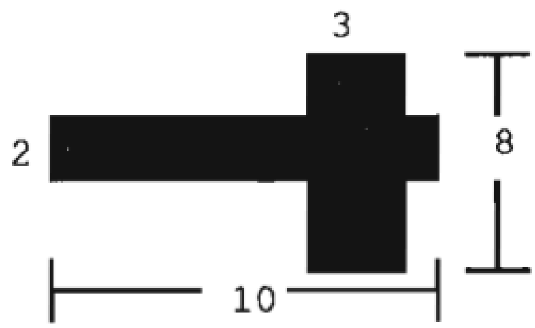
The volume is $7 \times 1 = 7$ cubic units. There are five unit squares facing each of six directions: front, back, top, bottom, left and right, for a total of 30 square units of surface area. The ratio of the volume to the surface area is $7 : 30$.

1988 Q17

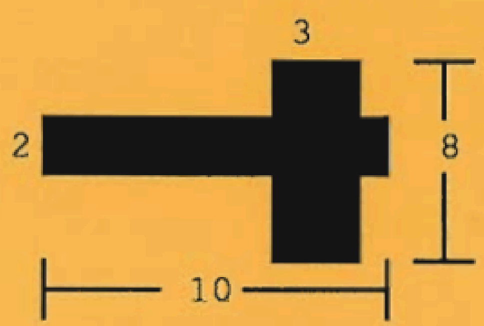
17. The shaded area formed by the two intersecting perpendicular rectangles, in square units, is

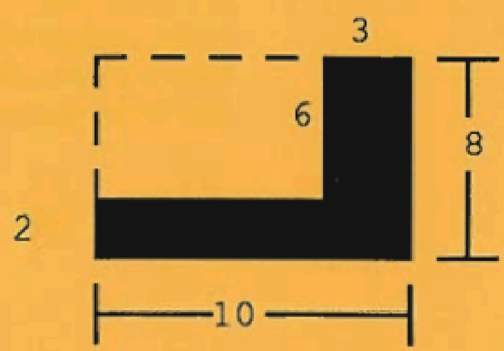
A) 23 B) 38 C) 44 D) 46

E) unable to be determined from the information given



17. B





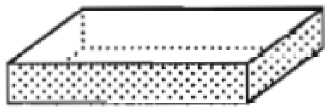
The total shaded area is the sum of the areas of the "horizontal" rectangle and the "vertical" rectangle minus the area of the "overlapping" rectangle that is part of both of the other rectangles. Thus the desired area is $2(10) + 3(8) - 2(3) = 38$.

OR

"Slide" the rectangles as shown in the figure on the right so they form an L-shaped figure. We see that the shaded area is $(10)(2) + (6)(3) = 38$ or $(7)(2) + (8)(3) = 38$ or $(10)(8) - (7)(6) = 38$.

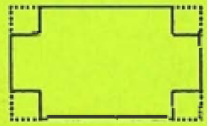
1993 Q17

17. Square corners, 5 units on a side, are removed from a 20 unit by 30 unit rectangular sheet of cardboard. The sides are then folded to form an open box. The surface area, in square units, of the interior of the box is



(A) 300 (B) 500 (C) 550 (D) 600 (E) 1000

17. (B) The interior (or exterior) has the same surface area as one side of the sheet of cardboard after the corners have been removed. The area of the sheet is $30 \times 20 = 600$ and the area of each of the square corners removed is $5 \times 5 = 25$, so the answer is $600 - (4 \times 25) = 500$.



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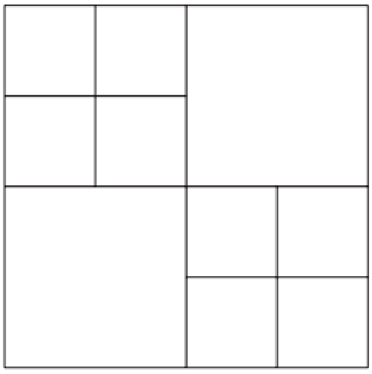
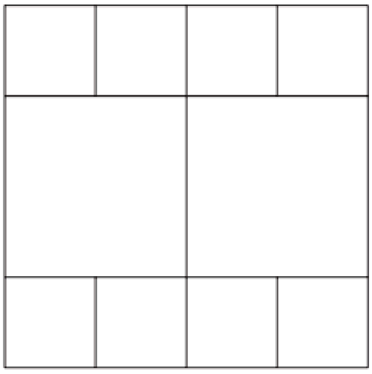
2012 Q17

17. A square with an integer side length is cut into 10 squares, all of which have integer side length and at least 8 of which have area 1. What is the smallest possible value of the length of the side of the original square?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

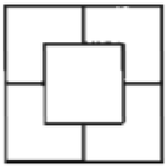
17. **Answer (B):** The area of the original square is a square number that is more than 8, so 16 is the least possible value for the area of the original square. Its side has length 4. Two possible ways of cutting the square are shown below:





1995 Q18

18. The area of each of the four congruent L-shaped regions of this 100-inch by 100-inch square is 3/16 of the total area. How many inches long is the side of the center square?

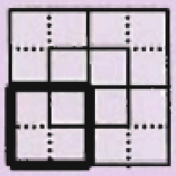


(A) 25 **(B)** 44 **(C)** 50 **(D)** 62 **(E)** 75

18. **(C)** The four L-shaped regions account for $4 \times \left(\frac{3}{16}\right) = \frac{12}{16} = \frac{3}{4}$ of the total area. That leaves $\frac{1}{4}$ of the total area for the center square, which yields $\left(\frac{1}{4}\right)(100 \times 100) = 2500$ square inches. Thus the length of the side of the center square is $\sqrt{2500} = 50$ inches.

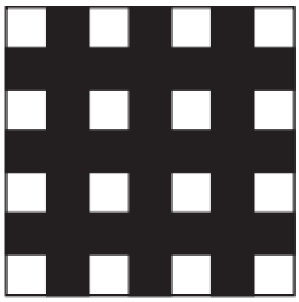
OR

As in the first solution, the center square is 1/4 of the total area. Dividing the large square into four equal parts as shown yields a square that is 50 by 50 inches.



2009 Q18

18. The diagram represents a 7-foot-by-7-foot floor that is tiled with 1-square-foot black tiles and white tiles. Notice that the corners have white tiles. If a 15-foot-by-15-foot floor is to be tiled in the same manner, how many white tiles will be needed?



(A) 49 **(B)** 57 **(C)** 64 **(D)** 96 **(E)** 126

18. **Answer (C):** To maintain the pattern, white squares will always occupy the corners, and every edge of the square pattern will have an odd number of tiles. Create a table, starting with a white square in the corner of the pattern, and increase the sides by 2 tiles.

Floor area	# of white squares	Pattern
1×1	1	1^2
3×3	4	2^2
5×5	9	3^2
7×7	16	4^2
9×9	25	5^2

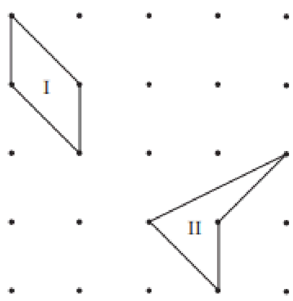
Following the pattern, an 11×11 area has 36 squares, a 13×13 area has 49, and a 15×15 has 64.

OR

There will be 8 rows that each contain 8 white tiles, so the total is $8(8) = 64$.

2000 Q18

18. Consider these two geoboard quadrilaterals. Which of the following statements is true?



(A) The area of quadrilateral I is more than the area of quadrilateral II.

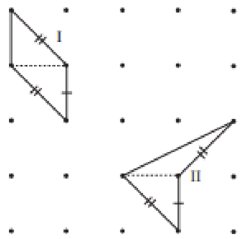
(B) The area of quadrilateral I is less than the area of quadrilateral II.

(C) The quadrilaterals have the same area and the same perimeter.

(D) The quadrilaterals have the same area, but the perimeter of I is more than the perimeter of II.

(E) The quadrilaterals have the same area, but the perimeter of I is less than the perimeter of II.

18. **Answer (E):** Divide each quadrilateral as shown. The resulting triangles each have base 1, altitude 1, and area $\frac{1}{2}$, so the quadrilaterals each have area 1.



Three sides of quadrilateral I match those of quadrilateral II as indicated by matching marks. The fourth side of quadrilateral I is less than the fourth side of quadrilateral II, hence its perimeter is less, and choice (E) is correct.

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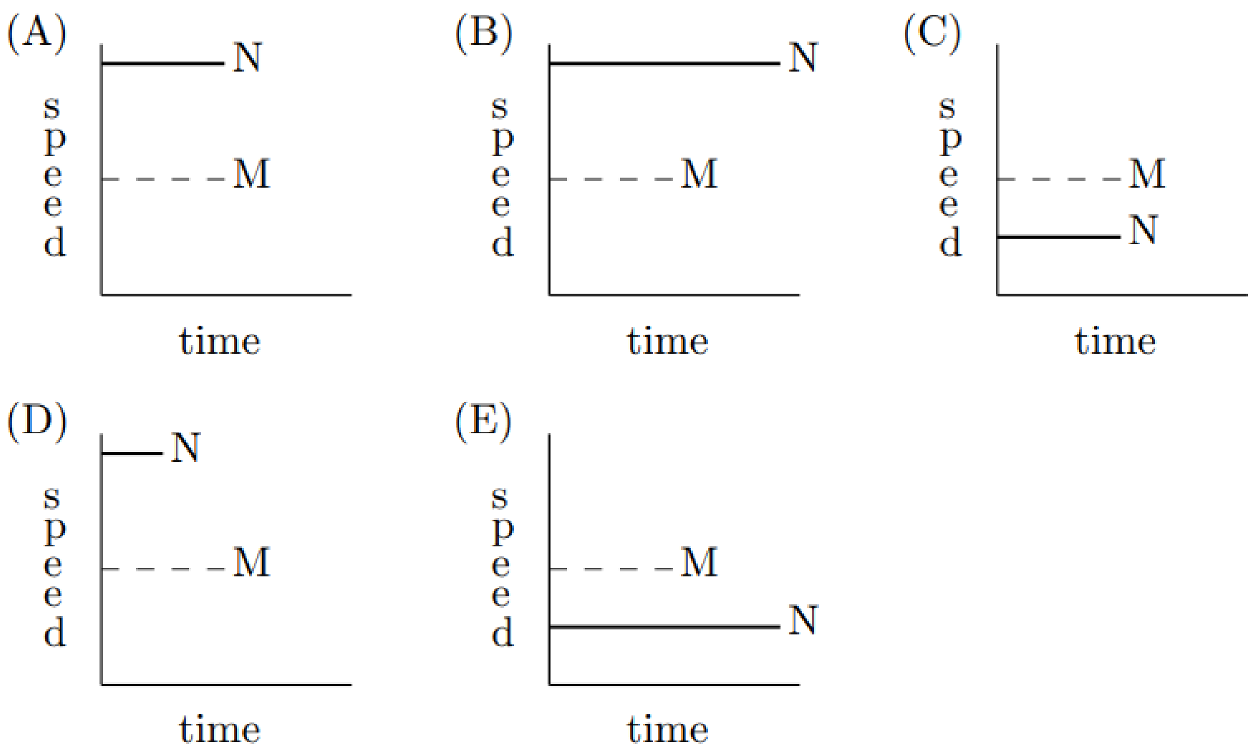
2001 Q19

19. Car M traveled at a constant speed for a given time. This is shown by the dashed line. Car N traveled at twice the speed for the same distance. If Car N's speed and time are shown as solid line, which graph illustrates this?



19. (D) The second car travels the same distance at twice the speed; therefore, it needs half the time required for the first car. Graph D shows this relationship.

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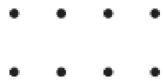
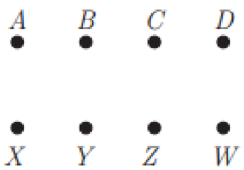
2009 Q20

20. How many non-congruent triangles have vertices at three of the eight points in the array shown below?



(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

20. **Answer (D):** With the points labeled as shown, one set of non-congruent triangles is AXY, AXZ, AXW, AYZ, AYW, AZW, BXZ and BXW.



Every other possible triangle is congruent to one of the 8 listed triangles.

CHALLENGE: Find the 48 distinct triangles possible and group them into sets of congruent triangles.