



# MAA AMC

American Mathematics Competitions

MAA American Mathematics Competitions

33<sup>rd</sup> Annual

# AMC 8

American Mathematics Competition 8  
Tuesday, November 14, 2017

## INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR COMPETITION MANAGER TELLS YOU.
2. This is a 25 question multiple choice test. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be scored.
4. There is no penalty for guessing. Your score is the number of correct answers.
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your competition manager will ask you to record your name and other information on the answer sheet.
8. You will have 40 minutes to complete the test once your competition manager tells you to begin.
9. When you finish the exam, *sign your name* in the space provided at the bottom of the answer sheet.

The MAA Committee on the American Mathematics Competitions reserves the right to disqualify scores from a school if it determines that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.



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The MAA American Mathematics Competitions are supported by:

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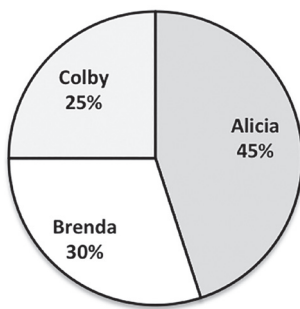
**Collaborator's Circle**

American Statistical Association  
Casualty Actuarial Society  
Conference Board of the Mathematical Sciences  
Mu Alpha Theta  
Society for Industrial and Applied Mathematics

1. Which of the following values is largest?

- (A)  $2 + 0 + 1 + 7$     (B)  $2 \times 0 + 1 + 7$     (C)  $2 + 0 \times 1 + 7$   
(D)  $2 + 0 + 1 \times 7$     (E)  $2 \times 0 \times 1 \times 7$

2. Alicia, Brenda, and Colby were the candidates in a recent election for student president. The pie chart below shows how the votes were distributed among the three candidates. If Brenda received 36 votes, then how many votes were cast all together?



- (A) 70    (B) 84    (C) 100    (D) 106    (E) 120

3. What is the value of the expression  $\sqrt{16\sqrt{8\sqrt{4}}}$ ?

- (A) 4    (B)  $4\sqrt{2}$     (C) 8    (D)  $8\sqrt{2}$     (E) 16

4. When 0.000315 is multiplied by 7,928,564 the product is closest to which of the following?

- (A) 210    (B) 240    (C) 2,100    (D) 2,400    (E) 24,000

5. What is the value of the expression  $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8}{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8}$ ?

- (A) 1020    (B) 1120    (C) 1220    (D) 2240    (E) 3360

6. If the degree measures of the angles of a triangle are in the ratio 3 : 3 : 4, what is the degree measure of the largest angle of the triangle?
- (A) 18    (B) 36    (C) 60    (D) 72    (E) 90
7. Let  $Z$  be a 6-digit positive integer, such as 247247, whose first three digits are the same as its last three digits taken in the same order. Which of the following numbers must be a factor of  $Z$  ?
- (A) 11    (B) 19    (C) 101    (D) 111    (E) 1111
8. Malcolm wants to visit Isabella after school today and knows the street where she lives but doesn't know her house number. She tells him, "My house number has two digits, and exactly three of the following four statements about it are true."

- (1) It is prime.  
(2) It is even.  
(3) It is divisible by 7.  
(4) One of its digits is 9.



This information allows Malcolm to determine Isabella's house number. What is its units digit?

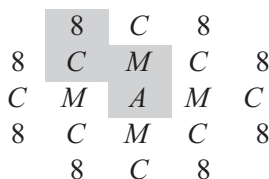
- (A) 4    (B) 6    (C) 7    (D) 8    (E) 9
9. All of Marcy's marbles are blue, red, green, or yellow. One third of her marbles are blue, one fourth of them are red, and six of them are green. What is the smallest number of yellow marbles that Marcy could have?
- (A) 1    (B) 2    (C) 3    (D) 4    (E) 5

10. A box contains five cards, numbered 1, 2, 3, 4, and 5. Three cards are selected randomly without replacement from the box. What is the probability that 4 is the largest value selected?
- (A)  $\frac{1}{10}$     (B)  $\frac{1}{5}$     (C)  $\frac{3}{10}$     (D)  $\frac{2}{5}$     (E)  $\frac{1}{2}$
11. A square-shaped floor is covered with congruent square tiles. If the total number of tiles that lie on the two diagonals is 37, how many tiles cover the floor?
- (A) 148    (B) 324    (C) 361    (D) 1296    (E) 1369
12. The smallest positive integer greater than 1 that leaves a remainder of 1 when divided by 4, 5, and 6 lies between which of the following pairs of numbers?
- (A) 2 and 19    (B) 20 and 39    (C) 40 and 59  
(D) 60 and 79    (E) 80 and 124
13. Peter, Emma, and Kyler played chess with each other. Peter won 4 games and lost 2 games. Emma won 3 games and lost 3 games. If Kyler lost 3 games, how many games did he win?



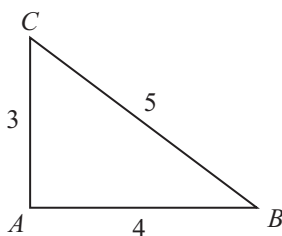
- (A) 0    (B) 1    (C) 2    (D) 3    (E) 4
14. Chloe and Zoe are both students in Ms. Demeanor's math class. Last night they each solved half of the problems in their homework assignment alone and then solved the other half together. Chloe had correct answers to only 80% of the problems she solved alone, but overall 88% of her answers were correct. Zoe had correct answers to 90% of the problems she solved alone. What was Zoe's overall percentage of correct answers?
- (A) 89    (B) 92    (C) 93    (D) 96    (E) 98

15. In the arrangement of letters and numerals below, by how many different paths can one spell  $AMC8$ ? Beginning at the  $A$  in the middle, a path allows only moves from one letter to an adjacent (above, below, left, or right, but not diagonal) letter. One example of such a path is traced in the picture.



- (A) 8    (B) 9    (C) 12    (D) 24    (E) 36

16. In the figure shown below, choose point  $D$  on side  $\overline{BC}$  so that  $\triangle ACD$  and  $\triangle ABD$  have equal perimeters. What is the area of  $\triangle ABD$ ?



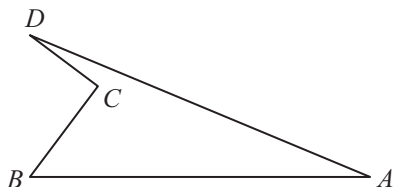
- (A)  $\frac{3}{4}$     (B)  $\frac{3}{2}$     (C) 2    (D)  $\frac{12}{5}$     (E)  $\frac{5}{2}$

17. Starting with some gold coins and some empty treasure chests, I tried to put 9 gold coins in each treasure chest, but that left 2 treasure chests empty. So instead I put 6 gold coins in each treasure chest, but then I had 3 gold coins left over. How many gold coins did I have?



- (A) 9    (B) 27    (C) 45    (D) 63    (E) 81

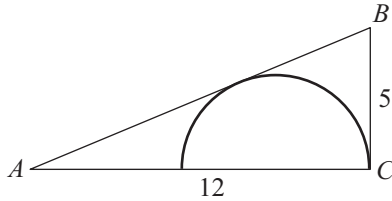
18. In the non-convex quadrilateral  $ABCD$  shown below,  $\angle BCD$  is a right angle,  $AB = 12$ ,  $BC = 4$ ,  $CD = 3$ , and  $AD = 13$ .



What is the area of quadrilateral  $ABCD$ ?

- (A) 12    (B) 24    (C) 26    (D) 30    (E) 36
19. For any positive integer  $M$ , the notation  $M!$  denotes the product of the integers 1 through  $M$ . What is the largest integer  $n$  for which  $5^n$  is a factor of the sum  $98! + 99! + 100!$  ?
- (A) 23    (B) 24    (C) 25    (D) 26    (E) 27
20. An integer between 1000 and 9999, inclusive, is chosen at random. What is the probability that it is an odd integer whose digits are all distinct?
- (A)  $\frac{14}{75}$     (B)  $\frac{56}{225}$     (C)  $\frac{107}{400}$     (D)  $\frac{7}{25}$     (E)  $\frac{9}{25}$
21. Suppose  $a$ ,  $b$ , and  $c$  are nonzero real numbers, and  $a + b + c = 0$ . What are the possible value(s) for  $\frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|}$ ?
- (A) 0    (B) 1 and  $-1$     (C) 2 and  $-2$     (D) 0, 2, and  $-2$   
 (E) 0, 1, and  $-1$

22. In the right triangle  $ABC$ ,  $AC = 12$ ,  $BC = 5$ , and angle  $C$  is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?



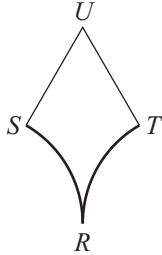
- (A)  $\frac{7}{6}$     (B)  $\frac{13}{5}$     (C)  $\frac{59}{18}$     (D)  $\frac{10}{3}$     (E)  $\frac{60}{13}$
23. Each day for four days, Linda traveled for one hour at a speed that resulted in her traveling one mile in an integer number of minutes. Each day after the first, her speed decreased so that the number of minutes to travel one mile increased by 5 minutes over the preceding day. Each of the four days, her distance traveled was also an integer number of miles. What was the total number of miles for the four trips?
- (A) 10    (B) 15    (C) 25    (D) 50    (E) 82
24. Mrs. Sanders has three grandchildren, who call her regularly. One calls her every three days, one calls her every four days, and one calls her every five days. All three called her on December 31, 2016. On how many days during the next year did she not receive a phone call from any of her grandchildren?



- (A) 78    (B) 80    (C) 144    (D) 146    (E) 152



25. In the figure shown,  $\overline{US}$  and  $\overline{UT}$  are line segments each of length 2, and  $m\angle TUS = 60^\circ$ . Arcs  $\widehat{TR}$  and  $\widehat{SR}$  are each one-sixth of a circle with radius 2. What is the area of the region shown?



- (A)  $3\sqrt{3} - \pi$       (B)  $4\sqrt{3} - \frac{4\pi}{3}$       (C)  $2\sqrt{3}$       (D)  $4\sqrt{3} - \frac{2\pi}{3}$   
 (E)  $4 + \frac{4\pi}{3}$



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## **How will I receive my score?**

Scores and solutions will be sent to your competition manager who can share that information with you. Use the solutions to learn more mathematics and enhance your problem-solving skills!

## **Are there more math competitions that I can participate in?**

The MAA American Mathematics Competitions also offers two high school level exams that are open to younger participants. These are both 25 question, 75-minute, multiple choice mathematics exams designed to promote the development of problem-solving skills. For more information visit [maa.org/amc](http://maa.org/amc).

## **How can I prepare for future math competitions?**

The best way to prepare for the MAA American Mathematics Competitions is to practice creative, analytical thinking throughout the year. Schools involved with the MAA AMC often have year-round activities connected to special classes, math clubs, or other extracurricular groups. Individual students can benefit greatly from practicing math problems from past MAA AMC exams.

## **Questions?**

Questions and comments about problems and solutions for this exam should be sent to:

[amchq@maa.org](mailto:amchq@maa.org)

Send questions and comments about administrative arrangements to:

[amcinfo@maa.org](mailto:amcinfo@maa.org)

or

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