



MATHEMATICAL ASSOCIATION OF AMERICA

MAA100

American Mathematics Competitions

30th Annual

AMC 8

American Mathematics Contest 8

Tuesday, November 18, 2014



INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score is the number of correct answers.
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record your information on the answer form.
8. You will have 40 minutes to complete the test once your proctor tells you to begin.
9. When you finish the exam, *sign your name* in the space provided on the answer form.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it determines that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of the AMC 8 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, email, internet or media of any type during this period is a violation of the competition rules. After the contest period, permission to make copies of individual problems in paper or electronic form including posting on web pages for educational use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear the copyright notice.

2014

AMC 8



DO NOT OPEN UNTIL TUESDAY, NOVEMBER 18, 2014

****ADMINISTRATION ON AN EARLIER DATE
WILL DISQUALIFY YOUR SCHOOL'S RESULTS****

1. PLEASE READ THE TEACHERS' MANUAL BEFORE NOVEMBER 18, 2014. All rules and instructions needed to administer this exam are contained in the manual. You will not need anything from inside this package until November 18.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 8 CERTIFICATION FORM that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be sent by trackable mail to the AMC office no later than 24 hours following the exam.
4. THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.
5. The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, email, internet or media of any type during this period is a violation of the competition rules.

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1. Harry and Terry are each told to calculate $8 - (2 + 5)$. Harry gets the correct answer. Terry ignores the parentheses and calculates $8 - 2 + 5$. If Harry's answer is H and Terry's answer is T , what is $H - T$?
- (A) -10 (B) -6 (C) 0 (D) 6 (E) 10
2. Paul owes Paula 35 cents and has a pocket full of 5-cent coins, 10-cent coins, and 25-cent coins that he can use to pay her. What is the difference between the largest and the smallest number of coins he can use to pay her?
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5



3. Isabella had a week to read a book for a school assignment. She read an average of 36 pages per day for the first three days and an average of 44 pages per day for the next three days. She then finished the book by reading 10 pages on the last day. How many pages were in the book?
- (A) 240 (B) 250 (C) 260 (D) 270 (E) 280



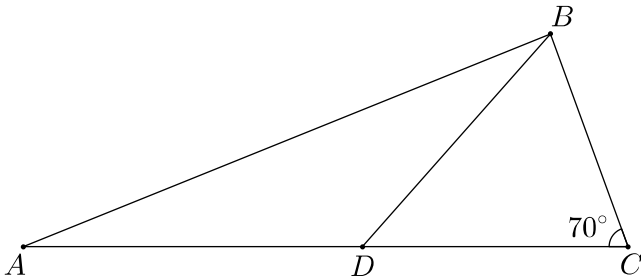
4. The sum of two prime numbers is 85. What is the product of these two prime numbers?
- (A) 85 (B) 91 (C) 115 (D) 133 (E) 166
5. Margie's car can go 32 miles on a gallon of gas, and gas currently costs \$4 per gallon. How many miles can Margie drive on \$20 worth of gas?
- (A) 64 (B) 128 (C) 160 (D) 320 (E) 640



6. Six rectangles each with a common base width of 2 have lengths of 1, 4, 9, 16, 25, and 36. What is the sum of the areas of the six rectangles?
- (A) 91 (B) 93 (C) 162 (D) 182 (E) 202
7. There are four more girls than boys in Ms. Raub's class of 28 students. What is the ratio of the number of girls to the number of boys in her class?
- (A) 3 : 4 (B) 4 : 3 (C) 3 : 2 (D) 7 : 4 (E) 2 : 1
8. Eleven members of the Middle School Math Club each paid the same amount for a guest speaker to talk about problem solving at their math club meeting. They paid their guest speaker $\$1\underline{A}2$. What is the missing digit A of this 3-digit number?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4



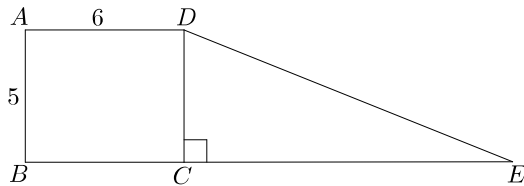
9. In $\triangle ABC$, D is a point on side \overline{AC} such that $BD = DC$ and $\angle BCD$ measures 70° . What is the degree measure of $\angle ADB$?
- (A) 100 (B) 120 (C) 135 (D) 140 (E) 150



10. The first AMC 8 was given in 1985 and it has been given annually since that time. Samantha turned 12 years old the year that she took the seventh AMC 8. In what year was Samantha born?
- (A) 1979 (B) 1980 (C) 1981 (D) 1982 (E) 1983
11. Jack wants to bike from his house to Jill's house, which is located three blocks east and two blocks north of Jack's house. After biking each block, Jack can continue either east or north, but he needs to avoid a dangerous intersection one block east and one block north of his house. In how many ways can he reach Jill's house by biking a total of five blocks?
- (A) 4 (B) 5 (C) 6 (D) 8 (E) 10

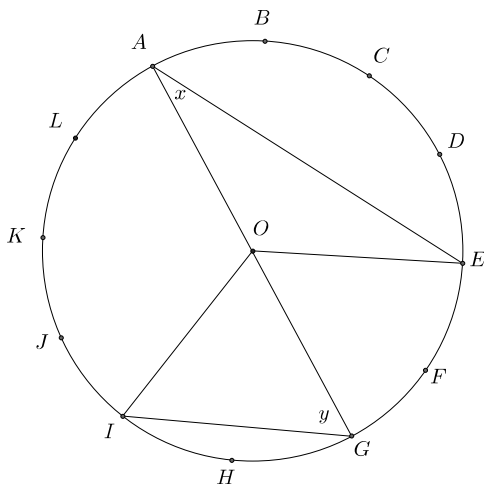


12. A magazine printed photos of three celebrities along with three photos of the celebrities as babies. The baby pictures did not identify the celebrities. Readers were asked to match each celebrity with the correct baby picture. What is the probability that a reader guessing at random will match all three correctly?
- (A) $\frac{1}{9}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$
13. If n and m are integers and $n^2 + m^2$ is even, which of the following is impossible?
- (A) n and m are even (B) n and m are odd (C) $n + m$ is even
 (D) $n + m$ is odd (E) none of these is impossible
14. Rectangle $ABCD$ and right triangle DCE have the same area. They are joined to form a trapezoid, as shown. What is DE ?
- (A) 12 (B) 13 (C) 14 (D) 15 (E) 16



15. The circumference of the circle with center O is divided into 12 equal arcs, marked the letters A through L as seen below. What is the number of degrees in the sum of angles x and y ?

(A) 75 (B) 80 (C) 90 (D) 120 (E) 150



16. The “Middle School Eight” basketball conference has 8 teams. Every season, each team plays every other conference team twice (home and away), and each team also plays 4 games against non-conference opponents. What is the total number of games in a season involving “Middle School Eight” teams?

(A) 60 (B) 88 (C) 96 (D) 144 (E) 160



17. George walks 1 mile to school. He leaves home at the same time each day, walks at a steady speed of 3 miles per hour, and arrives just as school begins. Today he was distracted by the pleasant weather and walked the first $\frac{1}{2}$ mile at a speed of only 2 miles per hour. At how many miles per hour must George run the last $\frac{1}{2}$ mile in order to arrive just as school begins today?

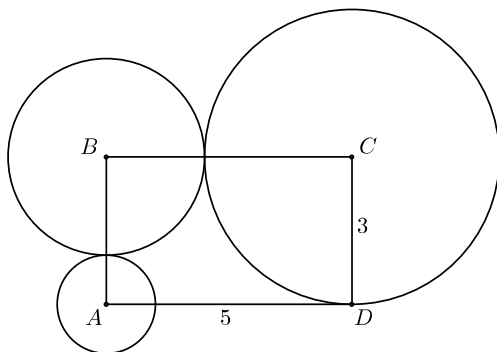
(A) 4 (B) 6 (C) 8 (D) 10 (E) 12



18. Four children were born at City Hospital yesterday. Assume each child is equally likely to be a boy or a girl. Which of the following outcomes is most likely?
- (A) all 4 are boys (B) all 4 are girls (C) 2 are girls and 2 are boys
 (D) 3 are of one gender and 1 is of the other gender
 (E) all of these outcomes are equally likely



19. A cube with 3-inch edges is to be constructed from 27 smaller cubes with 1-inch edges. Twenty-one of the cubes are colored red and 6 are colored white. If the 3-inch cube is constructed to have the smallest possible white surface area showing, what fraction of the surface area is white?
- (A) $\frac{5}{54}$ (B) $\frac{1}{9}$ (C) $\frac{5}{27}$ (D) $\frac{2}{9}$ (E) $\frac{1}{3}$
20. Rectangle $ABCD$ has sides $CD = 3$ and $DA = 5$. A circle of radius 1 is centered at A , a circle of radius 2 is centered at B , and a circle of radius 3 is centered at C . Which of the following is closest to the area of the region inside the rectangle but outside all three circles?
- (A) 3.5 (B) 4.0 (C) 4.5 (D) 5.0 (E) 5.5



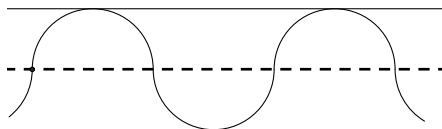
21. The 7-digit numbers $\overline{74A52B1}$ and $\overline{326AB4C}$ are each multiples of 3. Which of the following could be the value of C ?
- (A) 1 (B) 2 (C) 3 (D) 5 (E) 8
22. A 2-digit number is such that the product of the digits plus the sum of the digits is equal to the number. What is the units digit of the number?
- (A) 1 (B) 3 (C) 5 (D) 7 (E) 9
23. Three members of the Euclid Middle School girls' softball team had the following conversation.
- Ashley: I just realized that our uniform numbers are all 2-digit primes.
Bethany: And the sum of your two uniform numbers is the date of my birthday earlier this month.
Caitlin: That's funny. The sum of your two uniform numbers is the date of my birthday later this month.
Ashley: And the sum of your two uniform numbers is today's date.
What number does Caitlin wear?
- (A) 11 (B) 13 (C) 17 (D) 19 (E) 23
24. One day the Beverage Barn sold 252 cans of soda to 100 customers, and every customer bought at least one can of soda. What is the maximum possible median number of cans of soda bought per customer on that day?
- (A) 2.5 (B) 3.0 (C) 3.5 (D) 4.0 (E) 4.5



25. A straight one-mile stretch of highway, 40 feet wide, is closed. Robert rides his bike on a path composed of semicircles as shown. If he rides at 5 miles per hour, how many hours will it take to cover the one-mile stretch?

Note: 1 mile = 5280 feet

- (A) $\frac{\pi}{11}$ (B) $\frac{\pi}{10}$ (C) $\frac{\pi}{5}$ (D) $\frac{2\pi}{5}$ (E) $\frac{2\pi}{3}$





SOLUTIONS

Your School Manager will be sent at least one copy of the 2014 AMC 8 Solutions Pamphlet with the report. It is meant to be loaned to students (but not duplicated).

WRITE TO US

Comments about the problems and solutions for this AMC 8 should be addressed to:

Prof. Norbert Kuenzi, AMC 8 Chair
934 Nicolet Ave
Oshkosh, WI 54901-1634

Comments about administrative arrangements should be addressed to:
MAA American Mathematics Competitions / amcinfo@maa.org

American Mathematics Competitions
PO Box 471
Annapolis Junction, MD 20701

AMC 10 & AMC 12

The AMC 10 and AMC 12 are 25-question, 75-minute, multiple choice contests. All schools participating in the AMC 8 receive a brochure and registration form for the 2015 AMC 10. Schools with high scoring students on the AMC 8 should consider administering the AMC 10. The best way to prepare for these contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

American Mathematics Competitions
PO Box 471
Annapolis Junction, MD 20701

PUBLICATIONS

A complete listing of the current publications for sale can be found on our web site:
maa.org/math-competitions