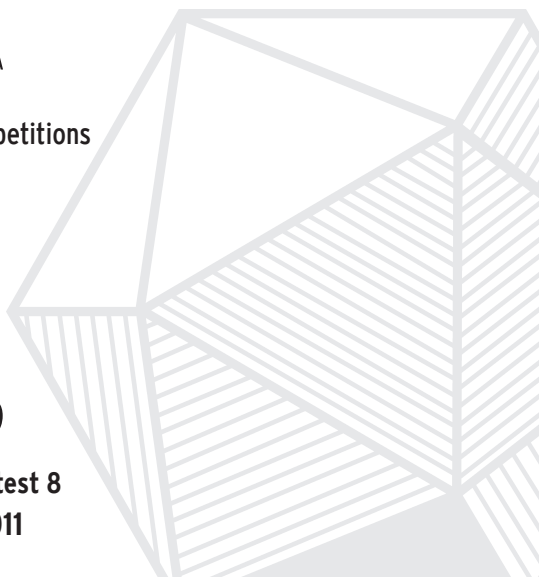**MAA**

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

27th Annual

AMC 8

American Mathematics Contest 8
Tuesday, November 15, 2011

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score is the number of correct answers.
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record your information on the answer form.
8. You will have 40 minutes to complete the test once your proctor tells you to begin.
9. When you finish the exam, *sign your name* in the space provided on the answer form.

2011
AMC 8

DO NOT OPEN UNTIL TUESDAY, NOVEMBER 15, 2011

**ADMINISTRATION ON AN EARLIER DATE
WILL DISQUALIFY YOUR SCHOOL'S RESULTS**

1. PLEASE READ THE TEACHERS' MANUAL BEFORE NOVEMBER 15, 2011. All rules and instructions needed to administer this exam are contained in the manual. You will not need anything from inside this package until November 15.

2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 8 CERTIFICATION FORM that you followed all rules associated with the conduct of the exam.

3. The Answer Forms must be sent by trackable mail to the AMC office no later than 24 hours following the exam.

4. THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.

5. The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, email, internet or media of any type during this period is a violation of the competition rules.

1. Margie bought 3 apples at a cost of 50 cents per apple. She paid with a 5-dollar bill. How much change did Margie receive?

    (A) $1.50   (B) $2.00   (C) $2.50   (D) $3.00   (E) $3.50

2. Karl's rectangular vegetable garden is 20 feet by 45 feet, and Makenna's is 25 feet by 40 feet. Whose garden is larger in area?

    (A) Karl's garden is larger by 100 square feet.

    (B) Karl's garden is larger by 25 square feet.

    (C) The gardens are the same size.

    (D) Makenna's garden is larger by 25 square feet.

    (E) Makenna's garden is larger by 100 square feet.

3. Extend the square pattern of 8 black and 17 white square tiles by attaching a border of black tiles around the square. What is the ratio of black tiles to white tiles in the extended pattern?
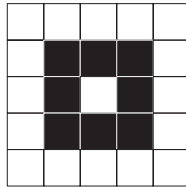
    (A) 8 : 17   (B) 25 : 49   (C) 36 : 25   (D) 32 : 17   (E) 36 : 17

4. Here is a list of the numbers of fish that Tyler caught in nine outings last summer:

$$2, 0, 1, 3, 0, 3, 3, 1, 2.$$

Which statement about the mean, mode, and median of these numbers is true?

    (A) median < mean < mode   (B) mean < mode < median

    (C) mean < median < mode   (D) median < mode < mean

    (E) mode < median < mean

5. What time was it 2011 minutes after the beginning of January 1, 2011?

(A) January 1 at 9:31PM

(B) January 1 at 11:51PM

(C) January 2 at 3:11AM

(D) January 2 at 9:31AM

(E) January 2 at 6:01PM

6. In a town of 351 adults, every adult owns a car, a motorcycle, or both. If 331 adults own cars, and 45 adults own motorcycles, how many of the car owners do not own a motorcycle?

(A) 20   (B) 25   (C) 45   (D) 306   (E) 351

7. Each of the following four large congruent squares is subdivided into combinations of congruent triangles or rectangles and is partially shaded. What percent of the total area is partially shaded?
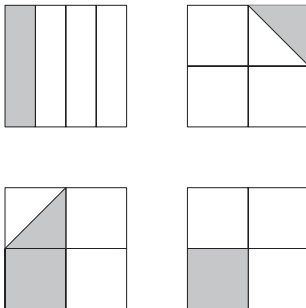
(A) $12\frac{1}{2}$ (B) 20   (C) 25   (D) $33\frac{1}{3}$ (E) $37\frac{1}{2}$

8. Bag A contains three chips labeled 1, 3, and 5. Bag B contains three chips labeled 2, 4, and 6. If one chip is drawn from each bag, how many different values are possible for the sum of the two numbers on the chips?

    (A) 4   **(B)** 5   **(C)** 6   **(D)** 7   **(E)** 9

9. Carmen takes a long bike ride on a hilly highway. The graph indicates the miles traveled during the time of her ride. What is Carmen's average speed for her entire ride in miles per hour?

H O U R S
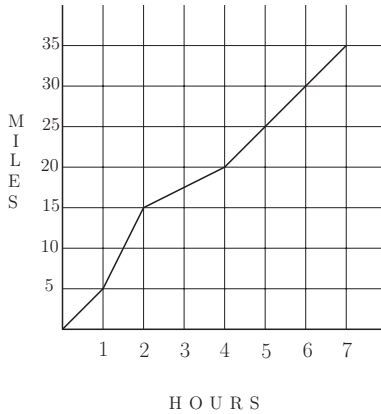
   **(A)** 2   **(B)** 2.5   **(C)** 4   **(D)** 4.5   **(E)** 5

10. The taxi fare in Gotham City is \$2.40 for the first $\frac{1}{2}$ mile and additional mileage charged at the rate \$0.20 for each additional 0.1 mile. You plan to give the driver a \$2 tip. How many miles can you ride for \$10 ?


TAXI

    (A) 3.0   **(B)** 3.25   **(C)** 3.3   **(D)** 3.5   **(E)** 3.75

11. The graph below shows the number of minutes studied by both Asha (black bar) and Sasha (grey bar) in one week. On the average, how many more minutes per day did Sasha study than Asha?
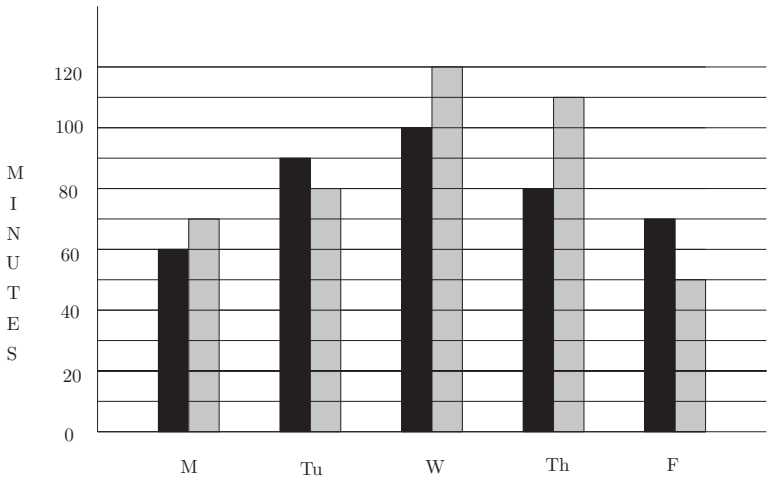
(A) 6   (B) 8   (C) 9   (D) 10   (E) 12

12. Angie, Bridget, Carlos, and Diego are seated at random around a square table, one person to a side. What is the probability that Angie and Carlos are seated opposite each other?

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{2}{3}$ (E) $\dfrac{3}{4}$

13. Two congruent squares, $ABCD$ and $PQRS$, have side length 15. They overlap to form the 15 by 25 rectangle $AQRD$ shown. What percent of the area of rectangle $AQRD$ is shaded?
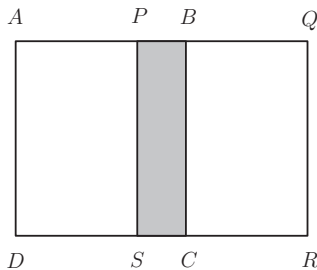
(A) 15   (B) 18   (C) 20   (D) 24   (E) 25

14. There are 270 students at Colfax Middle School, where the ratio of boys to girls is 5 : 4. There are 180 students at Winthrop Middle School, where the ratio of boys to girls is 4 : 5. The two schools hold a dance and all students from both schools attend. What fraction of the students at the dance are girls?

(A) $\dfrac{7}{18}$ (B) $\dfrac{7}{15}$ (C) $\dfrac{22}{45}$ (D) $\dfrac{1}{2}$ (E) $\dfrac{23}{45}$

15. How many digits are in the product $4^5 \cdot 5^{10}$?

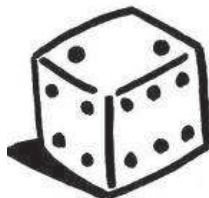(A) 8   (B) 9   (C) 10   (D) 11   (E) 15

16. Let A be the area of a triangle with sides of length 25, 25, and 30. Let B be the area of a triangle with sides of length 25, 25, and 40. What is the relationship between A and B?

(A) $A = \dfrac{9}{16}B$ (B) $A = \dfrac{3}{4}B$ (C) $A = B$ (D) $A = \dfrac{4}{3}B$ (E) $A = \dfrac{16}{9}B$

17. Let w, x, y, and z be whole numbers. If $2^w \cdot 3^x \cdot 5^y \cdot 7^z = 588$, then what does $2w + 3x + 5y + 7z$ equal?
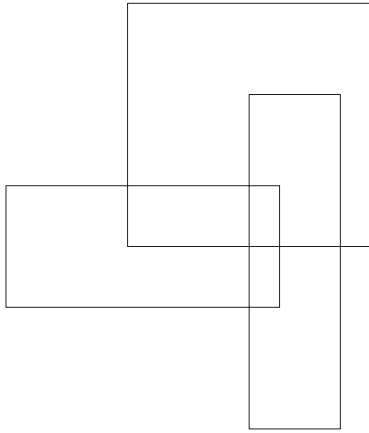
(A) 21   (B) 25   (C) 27   (D) 35   (E) 56

18. A fair six-sided die is rolled twice. What is the probability that the first number that comes up is greater than or equal to the second number?
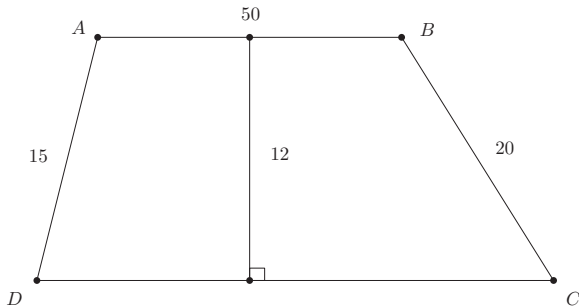

(A) $\dfrac{1}{6}$ (B) $\dfrac{5}{12}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{7}{12}$ (E) $\dfrac{5}{6}$

19. How many rectangles are in this figure?

(**A**) 8   (**B**) 9   (**C**) 10   (**D**) 11   (**E**) 12

20. Quadrilateral $ABCD$ is a trapezoid, $AD = 15$, $AB = 50$, $BC = 20$, and the altitude is 12. What is the area of the trapezoid?

(**A**) 600   (**B**) 650   (**C**) 700   (**D**) 750   (**E**) 800

21. Students guess that Norb's age is 24, 28, 30, 32, 36, 38, 41, 44, 47, and 49. Norb says, "At least half of you guessed too low, two of you are off by one and my age is a prime number." How old is Norb?

(**A**) 29   (**B**) 31   (**C**) 37   (**D**) 43   (**E**) 48

22. What is the **tens** digit of 7^{2011} ?

(**A**) 0   (**B**) 1   (**C**) 3   (**D**) 4   (**E**) 7

23. How many 4-digit positive integers have four different digits, where the leading digit is not zero, the integer is a multiple of 5, and 5 is the largest digit?

    (A) 24   **(B)** 48   **(C)** 60   **(D)** 84   **(E)** 108

24. In how many ways can 10,001 be written as the sum of two primes?

    (A) 0   **(B)** 1   **(C)** 2   **(D)** 3   **(E)** 4

25. A circle with radius 1 is inscribed in a square and circumscribed about another square as shown. Which fraction is closest to the ratio of the circle's shaded area to the shaded area between the two squares?
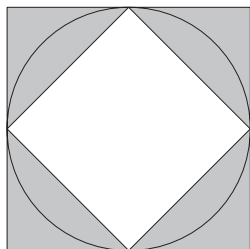
    (A) $\dfrac{1}{2}$ **(B)** 1   **(C)** $\dfrac{3}{2}$ **(D)** 2   **(E)** $\dfrac{5}{2}$

SOLUTIONS

Your School Manager has been sent at least one copy of the 2011 AMC 8 Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

WRITE TO US

Comments about the problems and solutions for this AMC 8 should be addressed to:
Dr. Margie Raub Hunt, AMC 8 Chair
2169 Madero Dr., The Villages, FL 32159

Comments about administrative arrangements should be addressed to:
MAA American Mathematics Competitions / amcinfo@maa.org
American Mathematics Competitions, University of Nebraska-Lincoln
P.O. Box 880658, Lincoln, NE 68588-0658

AMC 10 & AMC 12

The AMC 10 and AMC 12 are 25-question, 75-minute, multiple choice contests. All schools participating in the AMC 8 receive a brochure and registration form for the 2012 AMC 10. Schools with high scoring students on the AMC 8 should consider administering the AMC 10. The best way to prepare for these contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

American Mathematics Competitions
ATTN: Publications
P.O. Box 81606
Lincoln, NE 68501-1606

PUBLICATIONS

A complete listing of the current publications for sale can be found on our web site:
amc.maa.org