23rd Annual

AMC 8

(American Mathematics Contest 8)

Tuesday, NOVEMBER 13, 2007

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.

2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.

3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.

4. There is no penalty for guessing. Your score on this test is the number of correct answers.

5. No aids are permitted other than scratch paper, graph paper, rulers, erasers, and calculators that are accepted for use on the SAT. No problems on the test will require the use of a calculator.

6. Figures are not necessarily drawn to scale.

7. Before beginning the test, your proctor will ask you to record certain information on the answer form.

8. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to complete the test.

9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

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1. Theresa's parents have agreed to buy her tickets to see her favorite band if she spends an average of 10 hours per week helping around the house for 6 weeks. For the first 5 weeks she helps around the house for 8, 11, 7, 12 and 10 hours. How many hours must she work during the final week to earn the tickets?

   (A) 9   **(B)** 10   **(C)** 11   **(D)** 12   **(E)** 13
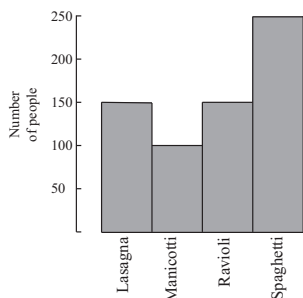
2. Six-hundred fifty students were surveyed about their pasta preferences. The choices were lasagna, manicotti, ravioli and spaghetti. The results of the survey are displayed in the bar graph. What is the ratio of the number of students who preferred spaghetti to the number of students who preferred manicotti?

   (A) $\dfrac{2}{5}$ **(B)** $\dfrac{1}{2}$ **(C)** $\dfrac{5}{4}$ **(D)** $\dfrac{5}{3}$ **(E)** $\dfrac{5}{2}$

3. What is the sum of the two smallest prime factors of 250?

   (A) 2   **(B)** 5   **(C)** 7   **(D)** 10   **(E)** 12

4. A haunted house has six windows. In how many ways can Georgie the Ghost enter the house by one window and leave by a different window?

   (A) 12   **(B)** 15   **(C)** 18   **(D)** 30   **(E)** 36

5. Chandler wants to buy a $500 mountain bike. For his birthday, his grandparents send him $50, his aunt sends him $35 and his cousin gives him $15. He earns $16 per week for his paper route. He will use all of his birthday money and all of the money he earns from his paper route. In how many weeks will he be able to buy the mountain bike?

   (A) 24   **(B)** 25   **(C)** 26   **(D)** 27   **(E)** 28

6. The average cost of a long-distance call in the USA in 1985 was 41 cents per minute, and the average cost of a long-distance call in the USA in 2005 was 7 cents per minute. Find the approximate percent decrease in the cost per minute of a long-distance call.
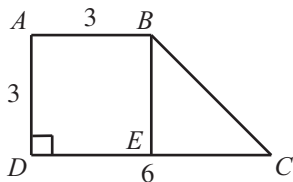
   (A) 7   **(B)** 17   **(C)** 34   **(D)** 41   **(E)** 80

7. The average age of 5 people in a room is 30 years. An 18-year-old person leaves the room. What is the average age of the four remaining people?

(A) 25   (B) 26   (C) 29   (D) 33   (E) 36

8. In trapezoid $ABCD$, $\overline{AD}$ is perpendicular to $\overline{DC}$, $AD = AB = 3$, and $DC = 6$. In addition, E is on $\overline{DC}$, and $\overline{BE}$ is parallel to $\overline{AD}$. Find the area of $\triangle BEC$.

(A) 3   (B) 4.5   (C) 6   (D) 9   (E) 18

9. To complete the grid below, each of the digits 1 through 4 must occur once in each row and once in each column. What number will occupy the lower right-hand square?

1		2	
2	3		
		4	

(A) 1   (B) 2   (C) 3   (D) 4   (E) cannot be determined

10. For any positive integer n, define $\boxed{n}$ to be the sum of the positive factors of n. For example, $\boxed{6} = 1 + 2 + 3 + 6 = 12$. Find $\boxed{\boxed{11}}$.

(A) 13   (B) 20   (C) 24   (D) 28   (E) 30

11. Tiles I, II, III and IV are translated so one tile coincides with each of the rectangles A, B, C and D. In the final arrangement, the two numbers on any side common to two adjacent tiles must be the same. Which of the tiles is translated to Rectangle C?
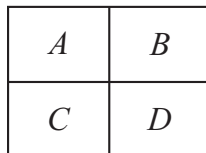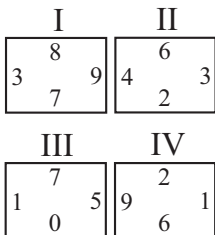
(A) I   (B) II   (C) III   (D) IV
(E) cannot be determined

12. A unit hexagram is composed of a regular hexagon of side length 1 and its 6 equilateral triangular extensions, as shown in the diagram. What is the ratio of the area of the extensions to the area of the original hexagon?
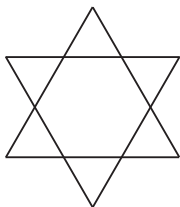

(A) 1:1 **(B)** 6:5 **(C)** 3:2 **(D)** 2:1 **(E)** 3:1

13. Sets A and B, shown in the Venn diagram, have the same number of elements. Their union has 2007 elements and their intersection has 1001 elements. Find the number of elements in A.
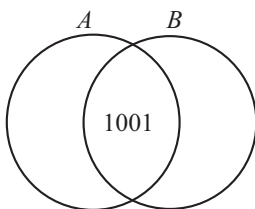
(A) 503 **(B)** 1006 **(C)** 1504 **(D)** 1507 **(E)** 1510

14. The base of isosceles $\triangle ABC$ is 24 and its area is 60. What is the length of one of the congruent sides?

(A) 5 **(B)** 8 **(C)** 13 **(D)** 14 **(E)** 18

15. Let a, b and c be numbers with $0 < a < b < c$. Which of the following is impossible?

(A) $a + c < b$ **(B)** $a \cdot b < c$ **(C)** $a + b < c$ **(D)** $a \cdot c < b$ **(E)** $\dfrac{b}{c} = a$

16. Amanda Reckonwith draws five circles with radii 1, 2, 3, 4 and 5. Then for each circle she plots the point (C, A), where C is its circumference and A is its area. Which of the following could be her graph?

(A)　　　　(B)　　　　(C)

(D)　　　　(E)
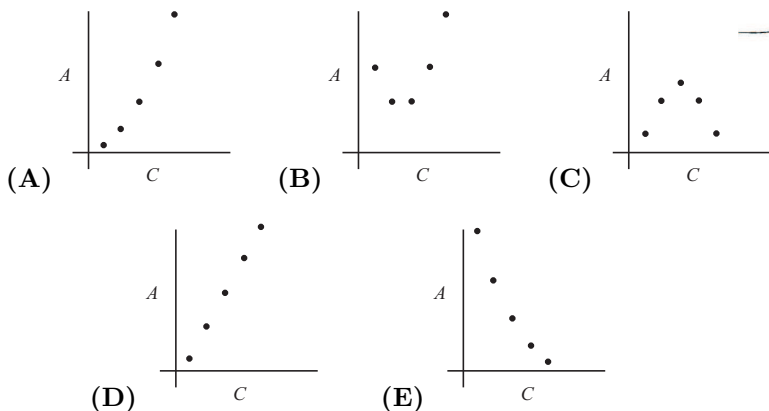
17. A mixture of 30 liters of paint is 25% red tint, 30% yellow tint and 45% water. Five liters of yellow tint are added to the original mixture. What is the percent of yellow tint in the new mixture?

(A) 25　　　(B) 35　　　(C) 40　　　(D) 45　　　(E) 50

18. The product of the two 99-digit numbers

$$303,030,303,\ldots,030,303 \quad \text{and} \quad 505,050,505,\ldots,050,505$$

has thousands digit A and units digit B. What is the sum of A and B?

(A) 3　　　(B) 5　　　(C) 6　　　(D) 8　　　(E) 10

19. Pick two consecutive positive integers whose sum is less than 100. Square both of those integers and then find the difference of the squares. Which of the following could be the difference?

(A) 2　　　(B) 64　　　(C) 79　　　(D) 96　　　(E) 131

20. Before district play, the Unicorns had won 45% of their basketball games. During district play, they won six more games and lost two, to finish the season having won half their games. How many games did the Unicorns play in all?

   **(A)** 48 **(B)** 50 **(C)** 52 **(D)** 54 **(E)** 60

21. Two cards are dealt from a deck of four red cards labeled A, B, C, D and four green cards labeled A, B, C, D. A winning pair is two of the same color or two of the same letter. What is the probability of drawing a winning pair?

    (A) $\dfrac{2}{7}$ **(B)** $\dfrac{3}{8}$ **(C)** $\dfrac{1}{2}$ **(D)** $\dfrac{4}{7}$ **(E)** $\dfrac{5}{8}$

22. A lemming sits at a corner of a square with side length 10 meters. The lemming runs 6.2 meters along a diagonal toward the opposite corner. It stops, makes a 90° right turn and runs 2 more meters. A scientist measures the shortest distance between the lemming and each side of the square. What is the average of these four distances in meters?

    (A) 2 **(B)** 4.5 **(C)** 5 **(D)** 6.2 **(E)** 7

23. What is the area of the shaded pinwheel shown in the 5×5 grid?
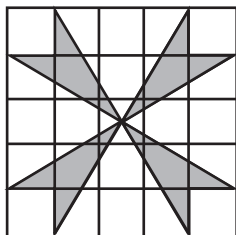

    (A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

24. A bag contains four pieces of paper, each labeled with one of the digits 1, 2, 3 or 4, with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of 3?

    (A) $\dfrac{1}{4}$ **(B)** $\dfrac{1}{3}$ **(C)** $\dfrac{1}{2}$ **(D)** $\dfrac{2}{3}$ **(E)** $\dfrac{3}{4}$

25. On the dart board shown in the figure, the outer circle has radius 6 and the inner circle has radius 3. Three radii divide each circle into three congruent regions, with point values shown. The probability that a dart will hit a given region is proportional to the area of the region. When two darts hit this board, the score is the sum of the point values in the regions. What is the probability that the score is odd?
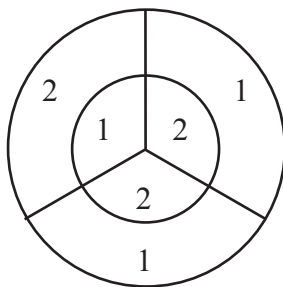
(A) $\dfrac{17}{36}$ (B) $\dfrac{35}{72}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{37}{72}$ (E) $\dfrac{19}{36}$

SOLUTIONS

Your School Manager has been sent at least one copy of the 2007 AMC 8 Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

WRITE TO US

Comments about the problems and solutions for this AMC 8 should be addressed to:
Ms. Bonnie Leitch, AMC 8 Chair / bleitch@earthlink.net
548 Hill Avenue, New Braunfels, TX 78130

Comments about administrative arrangements should be addressed to:
MAA American Mathematics Competitions / amcinfo@maa.org
American Mathematics Competitions, University of Nebraska-Lincoln
P.O. Box 880658, Lincoln, NE 68588-0658

AMC 10 & AMC 12

The AMC 10 and AMC 12 are 25-question, 75-minute contests with 5 choices of answers for each problem (A through E). Schools with high scoring students on the AMC 8 will receive an Invitation Brochure for the 2008 AMC 10. The best way to prepare for these upper level contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

American Mathematics Competitions
ATTN: Publications
P.O. Box 81606
Lincoln, NE 68501-1606

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.unl.edu/amc.

2007

AMC 8

DO NOT OPEN UNTIL
TUESDAY, NOVEMBER 13, 2007

Administration On An Earlier Date Will Disqualify Your School's Results

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE NOVEMBER 13, 2007. Nothing is needed from inside this package until November 13.

2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 8 CERTIFICATION FORM that you followed all rules associated with the conduct of the exam.

3. The Answer Forms must be mailed First Class to the AMC office no later than 24 hours following the exam.

4. THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.

5. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, e-mail, World Wide Web or media of any type is a violation of the competition rules.*