THE MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

21st Annual

AMC 8

(American Mathematics Contest 8)

Tuesday, NOVEMBER 15, 2005

INSTRUCTIONS

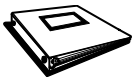
1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. No aids are permitted other than scratch paper, graph paper, rulers, erasers, and calculators that are accepted for use on the SAT. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it determines that the required security procedures were not followed.

1. Connie multiplies a number by 2 and gets 60 as her answer. However, she should have divided the number by 2 to get the correct answer. What is the correct answer?

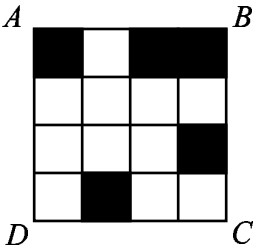
   (A) 7.5 **(B)** 15 **(C)** 30 **(D)** 120 **(E)** 240

2. Karl bought five folders from Pay-A-Lot at a cost of \$2.50 each. Pay-A-Lot had a 20%-off sale the following day. How much could Karl have saved on the purchase by waiting a day?



   (A) \$1.00 **(B)** \$2.00 **(C)** \$2.50 **(D)** \$2.75 **(E)** \$5.00

3. What is the minimum number of small squares that must be colored black so that a line of symmetry lies on the diagonal $\overline{BD}$ of square $ABCD$?



   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

4. A square and a triangle have equal perimeters. The lengths of the three sides of the triangle are 6.1 cm, 8.2 cm and 9.7 cm. What is the area of the square in square centimeters?

   (A) 24 **(B)** 25 **(C)** 36 **(D)** 48 **(E)** 64

5. Soda is sold in packs of 6, 12 and 24 cans. What is the minimum number of packs needed to buy exactly 90 cans of soda?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 15

6. Suppose d is a digit. For how many values of d is $2.00d5 > 2.005$?

   (A) 0 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 10

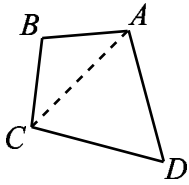
7. Bill walks $\frac{1}{2}$ mile south, then $\frac{3}{4}$ mile east, and finally $\frac{1}{2}$ mile south. How many miles is he, in a direct line, from his starting point?

   (A) 1 **(B)** $1\frac{1}{4}$ **(C)** $1\frac{1}{2}$ **(D)** $1\frac{3}{4}$ **(E)** 2

8. Suppose m and n are positive odd integers. Which of the following must also be an odd integer?

   (A) $m+3n$ **(B)** $3m-n$ **(C)** $3m^2+3n^2$ **(D)** $(nm+3)^2$ **(E)** $3mn$

9. In quadrilateral $ABCD$, sides $\overline{AB}$ and $\overline{BC}$ both have length 10, sides $\overline{CD}$ and $\overline{DA}$ both have length 17, and the measure of angle ADC is 60°. What is the length of diagonal $\overline{AC}$?



   (A) 13.5 **(B)** 14 **(C)** 15.5 **(D)** 17 **(E)** 18.5

10. Joe had walked half way from home to school when he realized he was late. He ran the rest of the way to school. He ran 3 times as fast as he walked. Joe took 6 minutes to walk half way to school. How many minutes did it take Joe to get from home to school?

    (A) 7 **(B)** 7.3 **(C)** 7.7 **(D)** 8 **(E)** 8.3

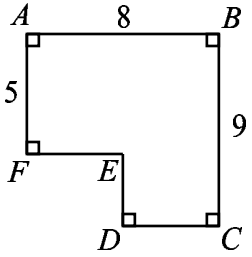
11. The sales tax rate in Bergville is 6%. During a sale at the Bergville Coat Closet, the price of a coat is discounted 20% from its \$90.00 price. Two clerks, Jack and Jill, calculate the bill independently. Jack rings up \$90.00 and adds 6% sales tax, then subtracts 20% from this total. Jill rings up \$90.00, subtracts 20% of the price, then adds 6% of the discounted price for sales tax. What is Jack's total minus Jill's total?

    (A) $-\$1.06$ **(B)** $-\$0.53$ **(C)** \$0 **(D)** \$0.53 **(E)** \$1.06

12. Big Al, the ape, ate 100 bananas from May 1 through May 5. Each day he ate six more bananas than on the previous day. How many bananas did Big Al eat on May 5?

    (A) 20 **(B)** 22 **(C)** 30 **(D)** 32 **(E)** 34

13. The area of polygon $ABCDEF$ is 52 with $AB = 8$, $BC = 9$ and $FA = 5$. What is $DE + EF$?



(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

14. The Little Twelve Basketball Conference has two divisions, with six teams in each division. Each team plays each of the other teams in its own division twice and every team in the other division once. How many conference games are scheduled?

(A) 80 **(B)** 96 **(C)** 100 **(D)** 108 **(E)** 192

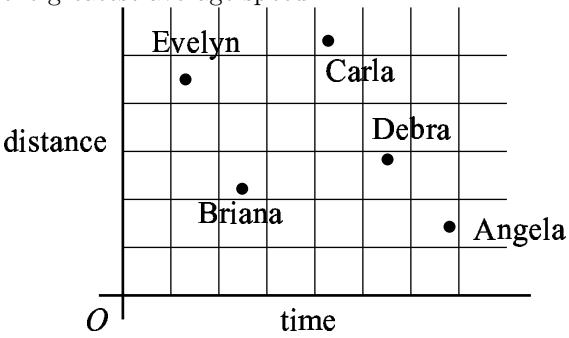
15. How many different isosceles triangles have integer side lengths and perimeter 23?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 9 **(E)** 11

16. A five-legged Martian has a drawer full of socks, each of which is red, white or blue, and there are at least five socks of each color. The Martian pulls out one sock at a time without looking. How many socks must the Martian remove from the drawer to be certain there will be 5 socks of the same color?

(A) 6 **(B)** 9 **(C)** 12 **(D)** 13 **(E)** 15

17. The results of a cross-country team's training run are graphed below. Which student has the greatest average speed?

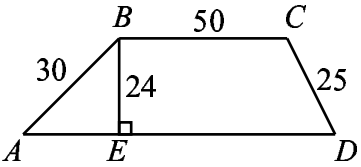


(A) Angela **(B)** Briana **(C)** Carla **(D)** Debra **(E)** Evelyn

18. How many three-digit numbers are divisible by 13?

(A) 7 **(B)** 67 **(C)** 69 **(D)** 76 **(E)** 77

19. What is the perimeter of trapezoid $ABCD$?

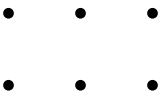


(A) 180 **(B)** 188 **(C)** 196 **(D)** 200 **(E)** 204

20. Alice and Bob play a game involving a circle whose circumference is divided by 12 equally-spaced points. The points are numbered clockwise, from 1 to 12. Both start on point 12. Alice moves clockwise and Bob, counterclockwise. In a turn of the game, Alice moves 5 points clockwise and Bob moves 9 points counterclockwise. The game ends when they stop on the same point. How many turns will this take?

(A) 6 **(B)** 8 **(C)** 12 **(D)** 14 **(E)** 24

21. How many distinct triangles can be drawn using three of the dots below as vertices?

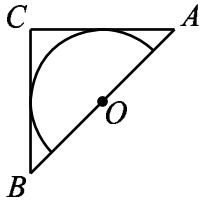


(A) 9 **(B)** 12 **(C)** 18 **(D)** 20 **(E)** 24

22. A company sells detergent in three different sized boxes: small (S), medium (M) and large (L). The medium size costs 50% more than the small size and contains 20% less detergent than the large size. The large size contains twice as much detergent as the small size and costs 30% more than the medium size. Rank the three sizes from best to worst buy.

(A) SML **(B)** LMS **(C)** MSL **(D)** LSM **(E)** MLS

23. Isosceles right triangle ABC encloses a semicircle of area 2π. The circle has its center O on hypotenuse $\overline{AB}$ and is tangent to sides $\overline{AC}$ and $\overline{BC}$. What is the area of triangle ABC?

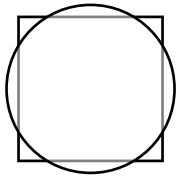


(A) 6 **(B)** 8 **(C)** 3π **(D)** 10 **(E)** 4π

24. A certain calculator has only two keys [+1] and [×2]. When you press one of the keys, the calculator automatically displays the result. For instance, if the calculator originally displayed "9" and you pressed [+1], it would display "10." If you then pressed [×2], it would display "20." Starting with the display "1," what is the fewest number of keystrokes you would need to reach "200"?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

25. A square with side length 2 and a circle share the same center. The total area of the regions that are inside the circle and outside the square is equal to the total area of the regions that are outside the circle and inside the square. What is the radius of the circle?



(A) $\frac{2}{\sqrt{\pi}}$ **(B)** $\frac{1+\sqrt{2}}{2}$ **(C)** $\frac{3}{2}$ **(D)** $\sqrt{3}$ **(E)** $\sqrt{\pi}$

SOLUTIONS

Your School Manager has been sent at least one copy of the 2005 AMC 8 Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

WRITE TO US

Comments about the problems and solutions for this AMC 8 should be addressed to:
Ms. Bonnie Leitch, AMC 8 Chair / bleitch@earthlink.net
548 Hill Avenue, New Braunfels, TX 78130

Comments about administrative arrangements should be addressed to:
MAA American Mathematics Competitions / amcinfo@unl.edu
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P.O. Box 880658, Lincoln, NE 68588-0658

AMC 10 & AMC 12

The AMC 10 and AMC 12 are 25-question, 75-minute contests with 5 choices of answers for each problem (A through E). Schools with high scoring students on the AMC 8 will receive an Invitation Brochure for the 2006 AMC 10. The best way to prepare for these upper level contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

American Mathematics Competitions
ATTN: Publications
P.O. Box 81606
Lincoln, NE 68501-1606

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: www.unl.edu/amc.

2005

AMC 8

DO NOT OPEN UNTIL TUESDAY, NOVEMBER 15, 2005

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE NOVEMBER 15, 2005. Nothing is needed from inside this package until November 15.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 8 CERTIFICATION FORM that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed First Class to the AMC office no later than 24 hours following the exam.
4. THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.
5. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, e-mail, World Wide Web or media of any type is a violation of the competition rules.*

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