Mathematical Association of America
American Mathematics Competitions
Presented by the Akamai Foundation

17th Annual

AMC 8

(American Mathematics Contest 8)

Solutions Pamphlet

Tuesday, NOVEMBER 13, 2001

This Solutions Pamphlet gives at least one solution for each problem on this year's exam and shows that all the problems can be solved using material normally associated with the mathematics curriculum for students in eighth grade or below. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

We hope that teachers will share these solutions with their students. However, the publication, reproduction, or communication of the problems or solutions of the AMC 8 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Duplication at any time via copier, telephone, e-mail, World Wide Web or media of any type is a violation of the copyright law.

Correspondence about the problems and solutions should be addressed to:
Prof. Joseph W. Kennedy, AMC 8 Chair / kennedj@muohio.edu
Department of Mathematics and Statistics, Miami University
Oxford, OH 45056

Orders for prior year Exam questions and Solutions Pamphlets should be addressed to:
Titu Andreescu, AMC Director / titu@amc.unl.edu
American Mathematics Competitions, University of Nebraska-Lincoln
P.O. Box 81606
Lincoln, NE 68501-1606

1. (D) At 2 seconds per dimple, it takes $300 \times 2 = 600$ seconds to paint them. Since there are 60 seconds in a minute, he will need $600 \div 60 = 10$ minutes.

2. (D) Since their sum is to be 11, only positive factors need to be considered. Number pairs whose product is 24 are $(1, 24)$, $(2, 12)$, $(3, 8)$ and $(4, 6)$. The sum of the third pair is 11, so the numbers are 3 and 8. The larger one is 8.

3. (E) Anjou has one-third as much money as Granny Smith, so Anjou has \$21. Elberta has \$2 more than Anjou, and $\$21 + \$2 = \$23$.

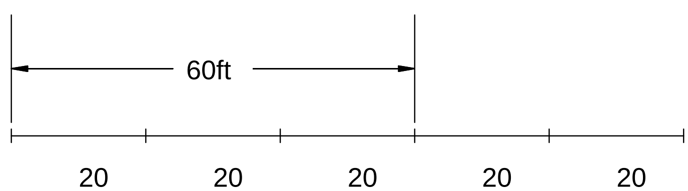
4. (E) To make the number as small as possible, the smaller digits are placed in the higher-value positions. To make the number even, the larger even digit 4 must be the units digit. The smallest possible even number is 12394 and 9 is in the tens place.

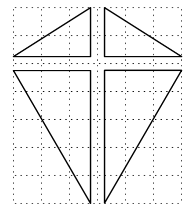
5. (C) Use the formula $d = rt$ (distance equals rate times time): 1088 feet per second $\times 10$ seconds $= 10880$ feet, which is just 320 feet more than two miles. Therefore, Snoopy is just about two miles from the flash of lightning.

   OR

   Since this is an estimate, round the speed of sound down to 1000 feet per second and the length of a mile down to 5000 feet. Then $5000 \div 1000 = 5$ seconds per mile, so in 10 seconds the sound will travel about 2 miles.

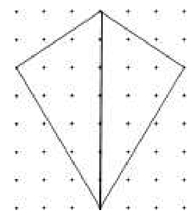
6. (B) There are three spaces between the first tree and the fourth tree, so the distance between adjacent trees is 20 feet. There are 6 trees with five of these 20-foot spaces, so the distance between the first and last trees is 100 feet.



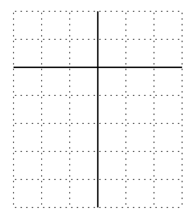


7. (A) The area is made up of two pairs of congruent triangles. The top two triangles can be arranged to form a 2×3 rectangle. The bottom two triangles can be arranged to form a 5×3 rectangle. The kite's area is $6 + 15 = 21$ square inches.

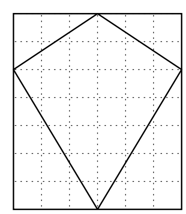
OR



The kite can be divided into two triangles, each with base 7 and altitude 3. Each area is $(1/2)(7)(3) = 10.5$, so the total area is $2(10.5) = 21$ square inches.



8. (E) The small kite is 6 inches wide and 7 inches high, so the larger kite is 18 inches wide and 21 inches high. The amount of bracing needed is $18 + 21 = 39$ inches.



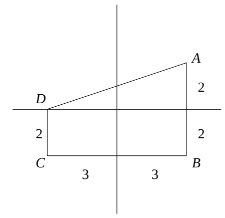
9. (D) The upper corners can be arranged to form a 6×9 rectangle and the lower corners can be arranged to form a 15×9 rectangle. The total area is $54 + 135 = 189$ square inches. (Note that the kite's area is also 189 square inches.)

OR

The area cut off equals the area of the kite. If each dimension is tripled, the area is $3 \times 3 = 9$ times as large as the original area and $21 \times 9 = 189$ square inches. In general, if one dimension is multiplied by a number x and the other by a number y, the area is multiplied by $x \times y$.

10. (A) $2000\% = 20.00$, so the quarters are worth 20 times their face value. That makes the total value $20(4)(\$0.25) = \$20$.

11. (C) The lower part is a 6×2 rectangle with area 12. The upper part is a triangle with base 6 and altitude 2 with area 6. The total area is $12 + 6 = 18$.



OR

Trapezoid $ABCD$ has bases 2 and 4 with altitude 6. Using the formula:

$$A = \frac{h(b_1 + b_2)}{2}, \text{ the area is } \frac{6(2+4)}{2} = 18.$$

12. (A) $6 \otimes 4 = \dfrac{6+4}{6-4} = \dfrac{10}{2} = 5$, and $5 \otimes 3 = \dfrac{5+3}{5-3} = \dfrac{8}{2} = 4$.

    Note: $(6 \otimes 4) \otimes 3 \neq 6 \otimes (4 \otimes 3)$. Does $(6 \otimes 4) \otimes 3 = 3 \otimes (6 \otimes 4)$?

13. (D) Since $12 + 8 + 6 = 26$, there are $36 - 26 = 10$ children who prefer cherry or lemon pie. These ten are divided into equal parts of 5 each.

$$\frac{5}{36} \times 360^\circ = 5 \times 10^\circ = 50^\circ.$$

14. (C) There are 3 choices for the meat and 4 for dessert.

    There are 6 ways to choose the two vegetables. The first vegetable may be chosen in 4 ways and the second in 3 ways. This would seem to make 12 ways, but since the order is not important the 12 must be divided by 2. Otherwise, for example, both tomatoes/corn and corn/tomatoes would be included. The 6 choices are beans/corn, beans/potatoes, beans/tomatoes, corn/potatoes, corn/tomatoes and potatoes/tomatoes.

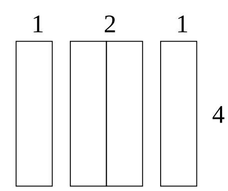
    The answer is 3(4)(6)=72.

15. (A) After 4 minutes Homer had peeled 12 potatoes. When Christen joined him, the combined rate of peeling was 8 potatoes per minute, so the remaining 32 potatoes required 4 minutes to peel. In these 4 minutes Christen peeled 20 potatoes.

OR

	minute	Homer	Christen	running total
	1	3		3
	2	3		6
	3	3		9
	4	3		12
	5	3	5	20
	6	3	5	28
	7	3	5	36
	8	3	5	44
Totals		24	20	

16. (E) The dimensions of the new rectangles are shown. The perimeter of a small rectangle is $4+1+4+1=10$ inches and for the large one it is $4+2+4+2=12$ inches. The ratio is $10/12=5/6$.



17. (B) The percent increase from a to b is given by

$$\frac{b-a}{a}(100\%)$$

For example, the percent increase for the first two questions is

$$\frac{200-100}{100}(100\%)=100\%$$

Each time the amount doubles there is a 100% increase. The only exceptions in this game are 2 to 3 (50%), 3 to 4 ($66\frac{2}{3}\%$) and 11 to 12 (about 95%). The answer is (B).

OR

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Value	100	200	300	500	1K	2K	4K	8K	16K	32K	64K	125K	250K	500K	1000K
% Increase		100	50	66.7	100	100	100	100	100	100	100	95	100	100	100

18. (D) There would be $6 \times 6 = 36$ entries in the table if it were complete, but only the 11 entries that are multiples of 5 are shown. The probability of getting a multiple of 5 is 11/36.

$\times$	1	2	3	4	5	6
1					5	
2					10	
3					15	
4					20	
5	5	10	15	20	25	30
6					30	

OR

Probability questions are sometimes answered by calculating the ways the event will NOT happen, then subtracting. In this problem the 1, 2, 3, 4 and 6 faces are paired to create $5 \times 5 = 25$ number pairs whose product is NOT multiples of 5. This leaves $36 - 25 = 11$ ways to get a multiple of 5, so the probability is 11/36.

19. (D) The second car travels the same distance at twice the speed; therefore, it needs half the time required for the first car. Graph D shows this relationship.

20. (A) Quay indicates that she has the same score as Kaleana. Marty's statement indicates that her score is higher than Kaleana's, and Shana's statement indicates that her score is lower than Kaleana's. The sequence S,Q,M is the correct one.

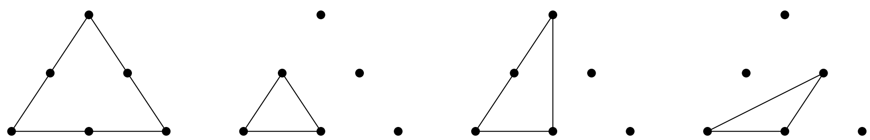
21. (D) The sum of all five numbers is 5(15)=75. Let the numbers be $W, X, 18, Y$ and Z in increasing order. For Z to be as large as possible, make W, X and Y as small as possible. The smallest possible values are $W = 1, X = 2$ and $Y = 19$. Then the sum of $W, X, 18$ and Y is 40, and the difference, $75 - 40 = 35$, is the largest possible value of Z.

22. (E) To get a score in the 90s, a student must get 18 or 19 correct answers. If the number is 18, then the other two questions are worth 0+0, 0+1, 1+0 or 1+1, producing total scores of 90, 91 or 92. If the number correct is 19, then the total is 95+0 or 95+1. Therefore, the only possible scores in the 90s are 90, 91, 92, 95 and 96. This leaves 97 as an impossible score.

OR

The highest possible score is 100 for 20 correct answers. For 19 correct the total is 95+0 or 95+1. This shows that 97 is not possible. As above, 90, 91 and 92 are possible.

23. (D) There are four noncongruent triangles.



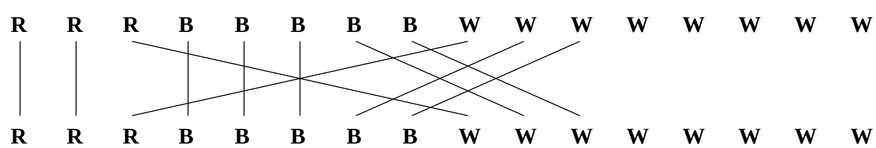
OR

The seventeen possible triangles may be divided into four congruence classes: {RST}; {RXY, XTZ, YZS, XYZ}; {RXS, TXS, RZS, RZT, TYR, TYS}; {RXZ, RYZ, TXY, TZY, XYS, XZS}

24. (B) All six red triangles are accounted for, so the two unmatched upper blue triangles must coincide with lower white triangles. Since one lower white triangle is matched with a red triangle and two are matched with blue triangles, there are five left and these must match with upper white triangles.

OR

It may be helpful to construct a diagram such as this:



25. (D) Six of the 24 numbers are in the 2000s, six in the 4000s, six in the 5000s and six in the 7000s. Doubling and tripling numbers in the 2000s produce possible solutions, but any multiple of those in the other sets is larger than 8000.

Units digits of the numbers are 2, 4, 5 and 7, so their doubles will end in 4, 8, 0 and 4, respectively. Choice (A) 5724 ends in 4 but $5724/2 = 2862$, not one of the 24 numbers. Likewise, choice (C) 7254 produces $7254/2 = 3627$, also not one of the numbers. When the units digits are tripled the resulting units digits are 6, 2, 5 and 1 and choices (B) 7245, (D) 7425 and (E) 7542 are possibilities. Division by 3 yields 2415, 2475 and 2514 respectively. Only the second of these numbers is one of the 24 given numbers. Choice (D) is correct.

The

American Mathematics Contest 8 (AMC 8)

Sponsored by

Mathematical Association of America

The Akamai Foundation

University of Nebraska – Lincoln

Contributors

American Mathematical Association of Two Year Colleges
American Mathematical Society
American Society of Pension Actuaries
American Statistical Association
Casualty Actuarial Society
Clay Mathematics Institute
Consortium for Mathematics and its Applications
Institute for Operations Research and the Management Sciences
Kappa Mu Epsilon
Mu Alpha Theta
National Association of Mathematicians
National Council of Teachers of Mathematics
Pi Mu Epsilon
School Science and Mathematics Association
Society of Actuaries