AMERICAN MATHEMATICS COMPETITIONS

15th ANNUAL

AMC $\rightarrow$ 8

(formerly AJHSME)

Solutions Pamphlet

TUESDAY, NOVEMBER 16, 1999

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This Solutions Pamphlet gives at least one solution for each problem on this year's exam and shows that all the problems can be solved using material normally associated with the mathematics curriculum for students in eighth grade or below. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

We hope that teachers will share these solutions with their students. *However, the publication, reproduction, or communication of the problems or solutions of the AMC → 8 during the period when students are eligible to participate seriously jeopardizes the integrity of the results.* Duplication at any time via copier, telephone, eMail, World Wide Web or media of any type is a violation of the copyright law.

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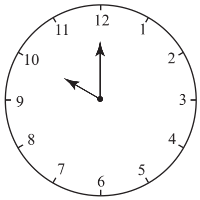
1. **Answer (A):**

$$(6\ ?\ 3) + 4 - (2 - 1) = 5$$
$$(6\ ?\ 3) + 4 - 1 = 5 \qquad \text{(subtract:}2 - 1 = 1)$$
$$(6\ ?\ 3) + 3 = 5 \qquad \text{(subtract:}4 - 1 = 3)$$
$$(6\ ?\ 3) = 2 \qquad \text{(subtract 3 from both sides)}$$
$$(6 \div 3) = 2$$

The other operations produce the following result:

$$(6 + 3) + 4 - (2 - 1) = 9 + 4 - 1 = 12$$
$$(6 - 3) + 4 - (2 - 1) = 3 + 4 - 1 = 6$$
$$(6 \times 3) + 4 - (2 - 1) = 18 + 4 - 1 = 21$$

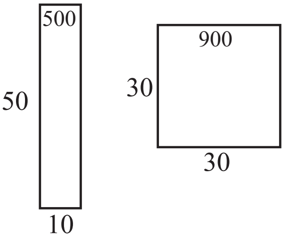
2. **Answer (C):** There are 360°(degrees) in a circle and twelve spaces on a clock. This means that each space measures 30°. At 10 o'clock the hands point to 10 and 12. They are two spaces or 60° apart.



3. **Answer (D):** $1.1 + (-2.1) + 1.0 = 0$. The other triplets add to 1 .

4. **Answer (A):** Four hours after starting, Alberto has gone about 60 miles and Bjorn has gone about 45 miles. Therefore, Alberto has biked about 15 more miles.

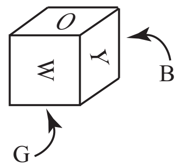
5. **Answer (D):** The area of the garden was 500 square feet(50×10) and its perimeter was 120 feet, $2 \times (50 + 10)$. The square garden is also enclosed by 120 feet of fence so its sides are each 30 feet long. The square garden's area is 900 square feet (30×30). and this has increased the garden area by 400 square feet.



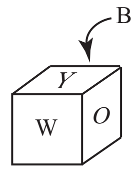
6. **Answer (E):** From the second sentence, Flo has more than someone so she can't have the least. From the third sentence both Bo and Coe have more than someone so that eliminates them. And, from the fourth sentence, Jo has more than someone, so that leaves only poor Moe!

7. **Answer (E):** There are $160 - 40 = 120$ miles between the third and tenth exits, so the service center is at milepost $40 + (3/4)120 = 40 + 90 = 130$.

8. **Answer (A):** When G is arranged to be the base, B is the back face and W is the front face. Thus, B is opposite W .

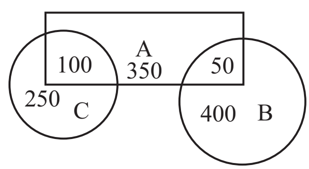


OR



Let Y be the top and fold G, O, and W down. Then B will fold to become the back face and be opposite W .

9. **Answer (C):** Bed A has 350 plants it doesn't share with B or C. Bed B has 400 plants it doesn't share with A or C. And C has 250 it doesn't share with A or B. The total is $350 + 400 + 250 + 50 + 100 = 1150$ plants.



OR

Plants shared by two beds have been counted twice, so the total is $500 + 450 + 350 - 50 - 100 = 1150$.

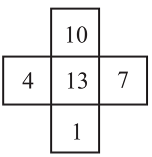
10. **Answer (E):**

$$\frac{\text{time not green}}{\text{total time}} = \frac{\text{R} + \text{Y}}{\text{R} + \text{Y} + \text{G}} = \frac{35}{60} = \frac{7}{12}.$$

OR

The probability of green is $\frac{25}{60} = \frac{5}{12}$. so the probability of not green is $1 - \frac{5}{12} = \frac{7}{12}$.

11. **Answer (D):** The largest sum occurs when 13 is placed in the center. This sum is $13 + 10 + 1 = 13 + 7 + 4 = 24$. Note: Two other common sums, 18 and 21, are possible.



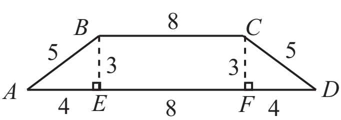
OR

Since the horizontal sum equals the vertical sum, twice this sum will be the sum of the five numbers plus the number in the center. When the center number is 13, the sum is the largest, $[10 + 4 + 1 + 7 + 2(13)]/2 = 48/2 = 24$. The other four numbers are divided into two pairs with equal sums.

12. **Answer (B):** The Won/Lost ratio is 11/4 so, for some number N, the team won $11N$ games and lost $4N$ games. Thus, the team played $15N$ games and the fraction of games lost is $\dfrac{4N}{15N} = \dfrac{4}{15} \approx 0.27 = 27\%$.

13. **Answer (C):** The sum of all ages is $40 \times 17 = 680$. The sum of the girls' ages is $20 \times 15 = 300$ and the sum of the boys' ages is $15 \times 16 = 240$. The sum of the five adults' ages is $680 - 300 - 240 = 140$. Therefore, their average is $\frac{140}{5} = 28$.

14. **Answer (D):** When the figure is divided, as shown the unknown sides are the hypotenuses of right triangles with legs of 3 and 4. Using the Pythagorean Theorem yields $AB = CD = 5$. The total perimeter is $16 + 5 + 8 + 5 = 34$.



15. **Answer (D):** Before new letters were added, five different letters could have been chosen for the first position, three for the second, and four for the third. This means that $5 \cdot 3 \cdot 4 = 60$ plates could have been made.
If two letters are added to the second set, then $5 \cdot 5 \cdot 4 = 100$ plates can be made. If one letter is added to each of the second and third sets, then $5 \cdot 4 \cdot 5 = 100$ plates can be made. None of the other four ways to place the two letters will create as many plates. So, $100 - 60 = 40$ ADDITIONAL plates can be made.

Note: Optimum results can usually be obtained in such problems by making the factors as nearly equal as possible.

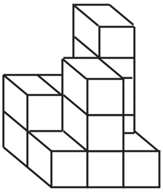
16. **Answer (B):** Since $70\%(10) + 40\%(30) + 60\%(35) = 7 + 12 + 21 = 40$, she answered 40 questions correctly. She needed $60\%(75) = 45$ to pass, so she needed 5 more correct answers.

17. **Answer (C):** One recipe makes 15 cookies, so $216 \div 15 = 14.4$ recipes are needed, but this must be rounded up to 15 recipes to make enough cookies. Each recipe requires 2 eggs. So 30 eggs are needed. This is 5 half-dozens.

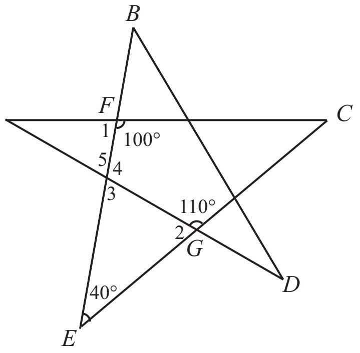
18. **Answer (E):** The $108(0.75) = 81$ students need 2 cookies each so 162 cookies are to be baked. Since $162 \div 15 = 10.8$, Walter and Gretel must bake 11 recipes. A few leftovers are a good thing!

19. **Answer (B):** Since $216 \div 15 = 14.4$, they will have to bake 15 recipes. This requires $15 \times 3 = 45$ tablespoons of butter. So, $45 \div 8 = 5.625$, and 6 sticks are needed.

20. **Answer (B):** The front view shows the larger of the numbers of cubes in the front or back stack in each column. Therefore the desired front view will have, from left to right, 2, 3, and 4 cubes. This is choice B.



21. **Answer (B):** Since $\angle 1$ forms a straight line with angle $100°$, $\angle 1 = 80°$. Since $\angle 2$ forms a straight line with angle $110°$, $\angle 2 = 70°$. Angle 3 is the third angle in a triangle with $\angle E = 40°$ and $\angle 2 = 70°$, so $\angle 3 = 180° - 40° - 70° = 70°$.Angle $4 = 110°$ since it forms a straight angle with $\angle 3$. Then $\angle 5$ forms a straight angle with $\angle 4$, so $\angle 5 = 70°$. (Or $\angle 3 = \angle 5$ because they are vertical angles.) Therefore, $\angle A = 180° - \angle 1 - \angle 5 = 180° - 80° - 70° = 30°$.



OR

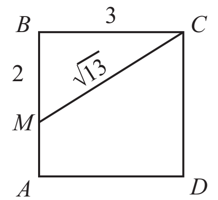
The angle sum in $\triangle CEF$ is $180°$, so $\angle C = 180° - 40° - 100° = 40°$. In $\triangle ACG$, $\angle G = 110°$ and $\angle C = 40°$, so $\angle A = 180° - 110° - 40° = 30°$.

22. **Answer (D):** One fish is worth $\frac{2}{3}$ of a loaf of bread and $\frac{2}{3}$ of a loaf of bread is worth $\frac{2}{3} \cdot 4 = \frac{8}{3} = 2\frac{2}{3}$ bags of rice.

OR

$$\begin{aligned} 3F &= 2B \\ \tfrac{3}{2}F &= B = 4R \\ (\tfrac{2}{3})(\tfrac{3}{2}) &= \tfrac{2}{3}(4R) \\ F &= \tfrac{8}{3}R = 2\tfrac{2}{3}R. \end{aligned}$$

23. **Answer (C):** One-third of the square's area is 3, so triangle MBC has area $3 = \frac{1}{2}(MB)(BC)$. Since side BC is 3, side MB must be 2. The hypotenuse CM of this right triangle is $\sqrt{2^2 + 3^2} = \sqrt{13}$.



24. **Answer (D):** Since any positive integer(expressed in base ten) is some multiple of 5 plus its last digit, its remainder when divided by 5 can be obtained by knowing its last digit.

Note that 1999^1 ends in 9, 1999^2 ends in 1, 1999^3 ends in 9, 1999^4 ends in 1, and this alternation of 9 and 1 endings continues with all even powers ending in 1. Therefore, the remainder when 1999^{2000} is divided by 5 is 1.

25. **Answer (A):** At each stage the area of the shaded triangle is one-third of the trapezoidal region not containing the smaller triangle being divided in the next step. Thus, the total area of the shaded triangles comes closer and closer to one-third of the area of the triangular region ACG and this is $\frac{1}{3} \cdot \frac{1}{2} \cdot 6 \cdot 6 = 6$. The shaded areas for the first six stages are: 4.5, 5.625, 5.906, 5.976, 5.994, and 5.998.

These are the calculations for the first three steps.

$\frac{1}{2} \cdot \frac{6}{2} \cdot \frac{6}{2} = 4.5$

$\frac{1}{2} \cdot \frac{6}{2} \cdot \frac{6}{2} + \frac{1}{2} \cdot \frac{6}{4} \cdot \frac{6}{4} = 4.5 + 1.125 = 5.625$

$\frac{1}{2} \cdot \frac{6}{2} \cdot \frac{6}{2} + \frac{1}{2} \cdot \frac{6}{4} \cdot \frac{6}{4} + \frac{1}{2} \cdot \frac{6}{8} \cdot \frac{6}{8} = 5.625 + 0.281 = 5.906$