

AMERICAN MATHEMATICS COMPETITIONS  
AJHSME SOLUTIONS PAMPHLET  
FOR STUDENTS AND TEACHERS  
14th ANNUAL  
AMERICAN JUNIOR HIGH SCHOOL  
MATHEMATICS EXAMINATION  
(AJHSME)  
Tuesday, NOVEMBER 17, 1998

*Sponsored by*

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This Solutions Pamphlet gives at least one solution for each problem on this year's Examination and shows that all the problems can be solved using material normally associated with the mathematics curriculum for students in eighth grade or below. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

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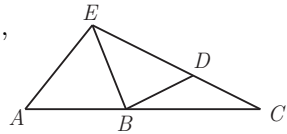
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1. **Answer (B):** Only  $\frac{6}{7}$ (A) and  $\frac{6}{8}$ (B) are less than 1. For fractions: If the numerators are equal, the smaller fraction will have a larger denominator. Therefore,  $\frac{6}{8}$  is smaller than  $\frac{6}{7}$ (A).

2. **Answer (E):**  $3 \cdot 2 - 4 \cdot 1 = 6 - 4 = 2$ .

3. **Answer (B):**  $\frac{3}{8} + \frac{7}{8} = \frac{10}{8} = \frac{5}{4}$ . Therefore,  $\frac{5}{4} \div \frac{4}{5} = \frac{5}{4} \cdot \frac{5}{4} = \frac{25}{16}$ .

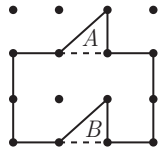
4. **Answer (E):** The triangles are  $ABE$ ,  $ACE$ ,  $BCD$ ,  $BCE$ ,  $BDE$ .



5. **Answer (B):**

- (A)  $9.12344 = 9.12344000\dots$ ,
- (B)  $9.123\bar{4} = 9.12344444\dots$ ,
- (C)  $9.12\bar{34} = 9.12343434\dots$ ,
- (D)  $9.\bar{1234} = 9.12342342\dots$ ,
- (E)  $9.\overline{1234} = 9.12341234\dots$

6. **Answer (B):** Slide triangle  $A$  down to fill in triangle  $B$ . The resulting  $2 \times 3$  rectangle has area 6.



7. **Answer (D):** Use the associative property to group as follows:  
 $(100 \times 19.98) \times (1.998 \times 1000) = 1998 \times 1998 = (1998)^2$ .

8. **Answer (C):**  $200\text{gallons} - 0.5(30)\text{gallons} = 200\text{gallons} - 15\text{gallons} = 185\text{gallons}$ .

9. **Answer (C):** First reduction:  $10 - 0.2(10) = 10 - 2 = 8$ .  
 Second reduction:  $8 - 0.5(8) = 8 - 4 = 4$ .

OR

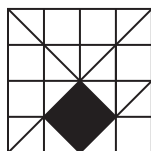
The sale price is 80% of the original price. After the next reduction, the final price is one-half the sale price, or one-half of 80% or 40%. Therefore,  $0.4(10) = 4$ .

10. **Answer (E):** The only arrangement that produces a whole number is  $\frac{3}{1} - \frac{4}{2} = 1$ .  
 Therefore,  $W + Y = 3 + 4 = 7$ .
11. **Answer (C):** Since Harry has 3 sisters and 5 brothers, there are 3 girls and 6 boys in the family. So Harriet has 2 sisters and 6 brothers. The product of 2 and 6 is 12.

12. **Answer (A):**

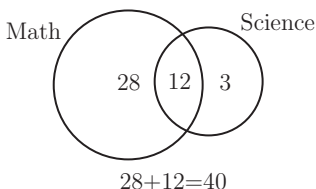
$$\begin{aligned} 2\left(1 - \frac{1}{2}\right) + 3\left(1 - \frac{1}{3}\right) + 4\left(1 - \frac{1}{4}\right) + \dots + 10\left(1 - \frac{1}{10}\right) &= \\ 2\left(\frac{1}{2}\right) + 3\left(\frac{2}{3}\right) + 4\left(\frac{3}{4}\right) + \dots + 10\left(\frac{9}{10}\right) &= \\ 1 + 2 + 3 + \dots + 9 &= 45. \end{aligned}$$

13. **Answer (C):** Divide the square into 16 smaller squares as shown. The shaded square is formed from 4 half-squares, so its area is 2. The ratio 2 to 16 is  $\frac{1}{8}$ .



Note: There are several other ways to divide the region to show this.

14. **Answer (E):** Since 80% of the Science Club members are also in the Math Club, there are  $0.8(15) = 12$  students common to both clubs. Because 30% of the students in the Math Club are also in the Science Club, there are  $12 \div 0.3 = 40$  students in the Math Club.



15. **Answer (D):**

In the year 1998 the population is 200.

In 2023 the population will be  $200(3) = 600$ .

In 2048 the population will be  $600(3) = 1800$ .

In 2050 the population will be about 2000.

16. **Answer (B):**

Year	Population
1998	200
2023	600
2048	1,800
2073	5,400
2098	16,200

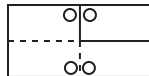
Of the choices available the year 2075 is the best estimate.

17. **Answer (C):**

Year	Population	Area Needed
1998	200	300
2023	600	900
2048	1,800	2,700
2073	5,400	8,100
2098	16,200	24,300

The Isles can support  $24,900 \div 1.5 = 16,600$  people. The chart shows that this will happen about the year 2098, or in about 100 years.

18. **Answer (B):** The folded rectangle appears in the upper right corner of the sheet of paper, and the hole is punched in its upper left corner. Only Figure (B) has a hole in the upper left corner of the upper right rectangle of the unfolded sheet.

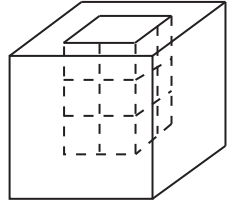


19. **Answer (A):** Tamika can get the numbers  $8 + 9 = 17$ ,  $8 + 10 = 18$ , or  $9 + 10 = 19$ . Carlos can get  $3 \times 5 = 15$ ,  $3 \times 6 = 18$ , or  $5 \times 6 = 30$ . The possible ways to pair these are:  $(17, 15)$ ,  $(17, 18)$ ,  $(17, 30)$ ,  $(18, 15)$ ,  $(18, 18)$ ,  $(18, 30)$ ,  $(19, 15)$ ,  $(19, 18)$ ,  $(19, 30)$ . Four of these nine pairs show Tamika with a higher result, so the probability is  $4/9$ .

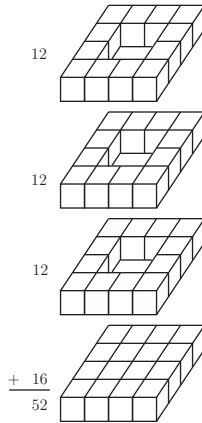
20. **Answer (D):** After folding the square twice the resulting figure is an isosceles triangle with area 9 square inches. Since there are 4 such congruent triangles in the square, the area of the square is 36 square inches. Therefore, the sides of  $PQRS$  are 6 inches, and the perimeter is 24 inches.



21. **Answer (B):** The  $2 \times 2 \times 3$  core contains all of the small cubes that do not touch a side or the bottom. These 12 cubes are subtracted from 64 to leave 52.



OR



22. **Answer (D):** The sequence is 98, 49, 44, 22, 11, 6, 54, 27, 22,  $\dots$ . After 3 terms the cycle (22, 11, 6, 54, 27) is repeated. The 98<sup>th</sup> term is the fifth term of the cycle, and this is 27.

23. **Answer (C):**

Step	Number of triangles	Number of shaded triangles
1	1	0
2	4	$0 + 1 = 1$
3	9	$1 + 2 = 3$
4	16	$1 + 2 + 3 = 6$
5	25	$1 + 2 + 3 + 4 = 10$
6	36	$1 + 2 + 3 + 4 + 5 = 15$
7	49	$1 + 2 + 3 + 4 + 5 + 6 = 21$
8	64	$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$

The ratio at step eight:  $\frac{28}{64} = \frac{7}{16}$ .

24. **Answer (E):** The numbers in the first column all have remainders of 1 when divided by 8, those of the second column have remainders of 2 when divided by 8, and so on. We need to find numbered squares so that each remainder 0 through 7 appears at least once. The squares that are shaded are numbered 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, and the remainders upon dividing by 8 are 1, 3, 6, 2, 7, 5, 4, 4, 5, 7, 2, 6, 3, 1, 0. Thus, we must shade square 120 to obtain the first shaded square in the last column.
25. **Answer (D):** Since Toy begins with \$36 and her amount is doubled in the first two exchanges, her amounts are \$36, \$72, \$144, and \$36. This means that she gave away \$108, and this is exactly enough to double the amounts of Ami and Jan. So, the total must be  $2(\$108) + \$36 = \$252$ .