AMERICAN MATHEMATICS COMPETITIONS

AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

13th ANNUAL

AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION

(AJHSME)

THURSDAY, NOVEMBER 20, 1997

Sponsored by

Mathematical Association of America
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This Solutions Pamphlet gives at least one solution for each problem on this year's Examination and shows that all the problems can be solved using material normally associated with the mathematics curriculum for students in eighth grade or below. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

PLEASE NOTE

We hope that teachers will share these solutions with their students. *However, the publication, reproduction, or communication of the problems or solutions of the AJHSME during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Duplication at any time via copier, telephone, eMail, World Wide Web or media of any type is a violation of the copyright law.*

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1. **(C)** In decimal form:

$$\begin{array}{r} 0.1 \\ 0.09 \\ 0.009 \\ +\ \ 0.0007 \\ \hline 0.1997 \end{array}$$

2. **(D)** The largest number is achieved by choosing the smallest number to subtract. The smallest two-digit number is 10, so the largest answer is (200-10) (2) = 380.

3. **(B)** Write each decimal to four places: and 0.9790 is seen to be the largest.

0.9700
0.9790
0.9709
0.9070
0.9089

4. **(E)** For one-half hour: 30 min. x 150 words/min. = 4500 words and for three-quarters of an hour: 45 min. x 150 words/min. = 6750 words. Only 5650 falls in this interval.

5. **(A)** The two-digit multiples of 7 are:

14, 21, (28), 35, 42, 49, 56, 63, 70, 77, 84, (91), and 98.

Only 28 and 91 have digit sums of 10. The sum of 28 and 91 is 119.

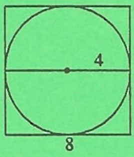
6. **(C)** Each shift of one place represents a multiple of 10:

7 4 9 8 2 1 0 3 5

100 10 1 0.1 0.01 0.001

Five shifts are needed and 10 x 10 x 10 x 10 x 10 = 10^5 = 100000.

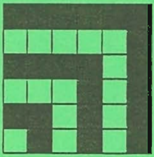
7. **(D)** The smallest square has side 8 and area $8^2 = 64$.



8. **(B)** Walter is gone from 7:30 a.m. until 4:00 p.m., a total of 8 hours and 30 minutes. He is in class for 6 (50 min.) = 300 min. or 5 hrs., at lunch for $\frac{1}{2}$ hr., and has 2 hours additional time. His total time at school is 7 hrs. and 30 min., so he was on the bus for 1 hour or 60 minutes.

9. **(C)** There are six ways to line up: ABC, ACB, BAC, BCA, CAB, CBA, and only one of these is in alphabetical order front-to-back. The probability is one in six or $\frac{1}{6}$.

10. **(C)** Of the 36 small squares, 21 are shaded, and $\frac{21}{36} = \frac{7}{12}$.



11. **(A)** Both 1 and 11 divide 11, so $\boxed{11} = 2$, and since 1, 2, 4, 5, 10, and 20 divide 20, then $\boxed{20} = 6$. The inner expression, $\boxed{11} \times \boxed{20} = 2 \times 6 = 12$. Finally, $\boxed{12} = 6$ because 1, 2, 3, 4, 6, and 12 divide 12.

12. **(D)** Since the sum of the angles of a triangle is 180°, $40° + 70° + \angle 1 = 180°$ and $\angle 1 = 70°$. This means that $\angle 2 = 110°$. Then $110° + \angle 3 + \angle 4 = 180°$, so $\angle 3 + \angle 4 = 70°$ and $\angle 3 = \angle 4 = 35°$.

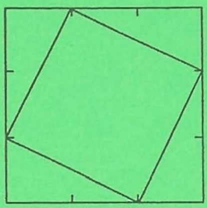
13. **(A)** There is a total of $26 + 28 + 30 = 84$ jelly beans:

$$\begin{aligned} 50\% \text{ of } 26 &= 13 \\ 25\% \text{ of } 28 &= 7 \\ 20\% \text{ of } 30 &= \underline{\ \ 6} \\ & \ \ \ 26 \end{aligned}$$

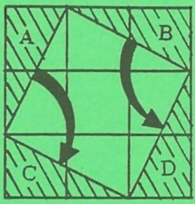
$$\frac{26}{84} = 0.3095 \approx 31\%$$

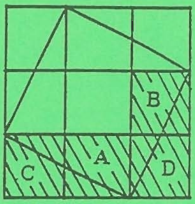
14. **(D)** Since 5 is the median, there must be two integers greater than 5 and two less than 5. The five integers may be arranged this way: ___ ___ 5 8 8. Since the mean is 5, the sum of all five integers is 25. The numbers 5, 8, and 8 total 21, leaving 4 for the sum of the first two. They can't both be 2 since 8 is the only mode, so they are 1 and 3. The difference between 8 and 1 is 7.

15. **(B)** Taking the side of the large square to be 3 inches gives an area of 9 square inches. Each of the four right triangles has an area of $\frac{1}{2}$ (2) (1) = 1 sq. inch. The area of the inscribed square is the area of the large square minus the area of the four right triangles, that is, 9 - 4 (1) = 5 sq. inches. The desired ratio is $\frac{5}{9}$.



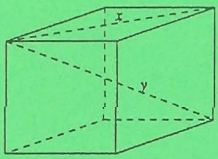
OR





16. **(E)** At the end of two years, stock values are:
Stock AA: \$100 (1.2) (0.8) = \$96
Stock BB: \$100 (0.75) (1.25) = \$93.75
Stock CC: \$100 (1) (1) = \$100
So, $B < A < C$.

17. **(E)** There are two diagonals, such as x, in each of the six faces for a total of twelve face diagonals. There are also four space diagonals, such as y, which are within the cube. This makes a total of 16.



18. **(B)** Last week one box cost \$1.25; this week one box costs \$0.80. The decrease, \$0.45, compared to the original price of \$1.25, is a decrease of $\frac{0.45}{1.25}$ = 0.36 or 36%, so choice (B) is closest.

OR

Last week there was a 4-box offer and this week a 5-box offer. Consider a purchase of 20 boxes (the smallest number divisible by both 4 and 5). Last week 20 boxes would have been \$25 (5 offers). This week it is \$16 (4 offers). The savings, \$9, compared to \$25 is 36%.

19. **(D)** Since $\frac{\cancel{3}}{2}\cdot\frac{\cancel{4}}{\cancel{3}}\cdot\frac{\cancel{5}}{\cancel{4}}\cdot\frac{\cancel{6}}{\cancel{5}}\cdot\ldots\cdot\frac{a}{\cancel{b}} = 9$, we see that $\frac{a}{2} = 9$. Thus, $a = 18$, $b = 17$, and $a + b = 35$.

20. **(A)** There are 64 equally likely possibilities for the numbers on the two dice. Of these, only (5, 8), (6, 7), (6, 8), (7, 6), (7, 7), (7, 8), (8, 5), (8, 6), (8, 7), and (8, 8) give products exceeding 36, so the probability of this occurring is $\frac{10}{64} = \frac{5}{32}$.

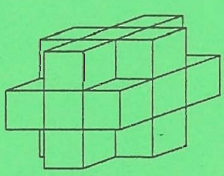
OR

Make a table for the sample space:

x	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32
5	5	10	15	20	25	30	35	(40)
6	6	12	18	24	30	36	(42)	(48)
7	7	14	21	28	35	(42)	(49)	(56)
8	8	16	24	32	(40)	(48)	(56)	(64)

The ten circled products exceed 36, so the probability of this occurring is $\frac{10}{64}$ or $\frac{5}{32}$.

21. **(D)** In terms of the original cube, three square centimeters are lost from each corner, but three new squares are added as the sides of the cavity in that corner. The total area remains at 54 square centimenters.



OR

Each face had an area of 9 square centimeters originally, so the total area was 6 (9) = 54 sq. cm. Four squares are lost from each face, leaving 54 - 4 (6) = 30 sq. cm. But the cavity in each of the eight corners has an area of 3 sq. cm, so 8 (3) = 24 sq. cm must be added. Finally, 30 + 24 = 54 sq. cm.

22. **(E)** The volume of a two-inch cube is $2^3 = 8$ cu. inches, while that of a three-inch cube is 27 cu. inches. Therefore, the weight and value of the larger cube is $\frac{27}{8}$ times that of the smaller. $\$200\left(\frac{27}{8}\right) = \$675$.

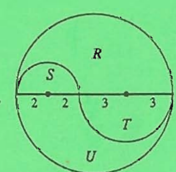
Note: The actual weight of the cubes is not needed to solve the problem.

23. **(C)** To meet the first condition, numbers which sum to 50 must be chosen from the set of squares $\{1, 4, 9, 16, 25, 36, 49\}$. To meet the second condition, the squares selected must be different. Consequently, there are three possibilities: $1 + 49$, $1 + 4 + 9 + 36$, and $9 + 16 + 25$. These correspond to the integers 17, 1236, and 345, respectively. The largest is 1236, and the product of its digits is $1 \cdot 2 \cdot 3 \cdot 6 = 36$.

24. **(C)** Let the diameter of the large circle equal 10. Then the ratio is:

$$\frac{\begin{pmatrix}\text{Area of}\\ \text{semicircle}\\ \text{R+S}\end{pmatrix} - \begin{pmatrix}\text{Area of}\\ \text{semicircle}\\ \text{S}\end{pmatrix} + \begin{pmatrix}\text{Area of}\\ \text{semicircle}\\ \text{T}\end{pmatrix}}{\begin{pmatrix}\text{Area of}\\ \text{semicircle}\\ \text{T+U}\end{pmatrix} - \begin{pmatrix}\text{Area of}\\ \text{semicircle}\\ \text{T}\end{pmatrix} + \begin{pmatrix}\text{Area of}\\ \text{semicircle}\\ \text{S}\end{pmatrix}}$$

$$\frac{\frac{1}{2}\pi 5^2 - \frac{1}{2}\pi 2^2 + \frac{1}{2}\pi 3^2}{\frac{1}{2}\pi 5^2 - \frac{1}{2}\pi 3^2 + \frac{1}{2}\pi 2^2} = \frac{15\pi}{10\pi} = \frac{3}{2} \text{ or } 3{:}2.$$



25. **(D)** If the numbers 2, 4, 6, and 8 are multiplied, the product is 384, so 4 is the final digit of the product of a set of numbers ending in 2, 4, 6, and 8. Since there are ten such sets of numbers, the final digit of the overall product is the same as the final digit of 4^{10}. Now, $4^{10} = (4^2)^5 = 16^5$. Next, consider 6^5. Since any number of 6's multiply to give 6 as the final digit, the final digit of the required product is 6.