AMERICAN MATHEMATICS COMPETITIONS

13th ANNUAL

AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION

(AJHSME)

THURSDAY, NOVEMBER 20, 1997

Sponsored by

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INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AJHSME ANSWER FORM with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. No aids are permitted other than scratch paper, graph paper, ruler, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working the problems. You will have **40 MINUTES** working time for the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

1. $\frac{1}{10} + \frac{9}{100} + \frac{9}{1000} + \frac{7}{10000} =$

(A) 0.0026 **(B)** 0.0197 **(C)** 0.1997 **(D)** 0.26 **(E)** 1.997

2. Ahn chooses a two-digit number, subtracts it from 200, and doubles the result. What is the largest number Ahn can get?

(A) 200 **(B)** 202 **(C)** 220 **(D)** 380 **(E)** 398

3. Which of the following numbers is the largest?

(A) 0.97 **(B)** 0.979 **(C)** 0.9709 **(D)** 0.907 **(E)** 0.9089

4. Julie is preparing a speech for her class. Her speech must last between one-half hour and three-quarters of an hour. The ideal rate of speech is 150 words per minute. If Julie speaks at the ideal rate, which of the following number of words would be an appropriate length for her speech?

(A) 2250 **(B)** 3000 **(C)** 4200 **(D)** 4350 **(E)** 5650

5. There are many two-digit multiples of 7, but only two of the multiples have a digit sum of 10. The sum of these two multiples of 7 is

(A) 119 **(B)** 126 **(C)** 140 **(D)** 175 **(E)** 189

6. In the number 74982.1035 the value of the *place* occupied by the digit 9 is how many times as great as the value of the *place* occupied by the digit 3?

(A) 1,000 **(B)** 10,000 **(C)** 100,000 **(D)** 1,000,000 **(E)** 10,000,000

7. The area of the smallest square that will contain a circle of radius 4 is

(A) 8 **(B)** 16 **(C)** 32 **(D)** 64 **(E)** 128

8. Walter gets up at 6:30 a.m., catches the school bus at 7:30 a.m., has 6 classes that last 50 minutes each, has 30 minutes for lunch, and has 2 hours additional time at school. He takes the bus home and arrives at 4:00 p.m. How many minutes has he spent on the bus?

(A) 30 **(B)** 60 **(C)** 75 **(D)** 90 **(E)** 120

9. Three students, with different names, line up single file. What is the probability that they are in alphabetical order from front-to-back?

(A) $\frac{1}{12}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{2}{3}$

10. What fraction of this square region is shaded? Stripes are equal in width, and the figure is drawn to scale.

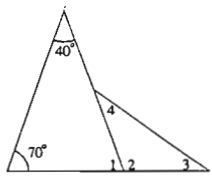
(A) $\frac{5}{12}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{7}{12}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{5}{6}$

11. Let $\boxed{N}$ mean the number of whole number divisors of N. For example, $\boxed{3} = 2$, because 3 has two divisors, 1 and 3. Find the value of

$$\boxed{\boxed{11} \times \boxed{20}}$$

(A) 6 **(B)** 8 **(C)** 12 **(D)** 16 **(E)** 24

12. $\angle 1 + \angle 2 = 180°$
$\angle 3 = \angle 4$
Find $\angle 4$



(A) 20° **(B)** 25° **(C)** 30° **(D)** 35° **(E)** 40°

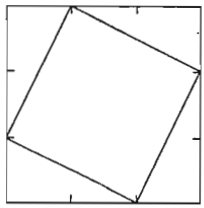
13. Three bags of jelly beans contain 26, 28, and 30 beans. The ratios of yellow beans to all beans in each of these bags are 50%, 25%, and 20%, respectively. All three bags of candy are dumped into one bowl. Which of the following is closest to the ratio of yellow jelly beans to all beans in the bowl?

(A) 31% **(B)** 32% **(C)** 33% **(D)** 35% **(E)** 95%

14. There is a set of five positive integers whose average (mean) is 5, whose median is 5, and whose only mode is 8. What is the difference between the largest and smallest integers in the set?

(A) 3 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

15. Each side of the large square in the figure is trisected (divided into three equal parts). The corners of an inscribed square are at these trisection points, as shown. The ratio of the area of the inscribed square to the area of the large square is

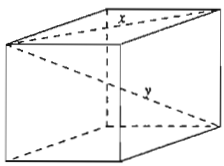


(A) $\frac{\sqrt{3}}{3}$ **(B)** $\frac{5}{9}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{\sqrt{5}}{3}$ **(E)** $\frac{7}{9}$

16. Penni Precisely buys \$100 worth of stock in each of three companies: Alabama Almonds, Boston Beans, and California Cauliflower. After one year, AA was up 20%, BB was down 25%, and CC was unchanged. For the second year, AA was down 20% from the previous year, BB was up 25% from the previous year, and CC was unchanged. If A, B, and C are the final values of the stock, then

(A) $A = B = C$ **(B)** $A = B < C$ **(C)** $C < B = A$
(D) $A < B < C$ **(E)** $B < A < C$

17. A cube has eight vertices (corners) and twelve edges. A segment, such as x, which joins two vertices not joined by an edge is called a diagonal. Segment y is also a diagonal. How many diagonals does a cube have?



(A) 6 **(B)** 8 **(C)** 12 **(D)** 14 **(E)** 16

18. At the grocery store last week, small boxes of facial tissue were priced at 4 boxes for \$5. This week they are on sale at 5 boxes for \$4. The percent decrease in the price per box during the sale was closest to

(A) 30% **(B)** 35% **(C)** 40% **(D)** 45% **(E)** 65%

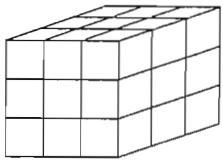
19. If the product $\frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdot \frac{6}{5} \cdot \ldots \cdot \frac{a}{b} = 9$, what is the sum of a and b?

(A) 11 **(B)** 13 **(C)** 17 **(D)** 35 **(E)** 37

20. A pair of 8-sided dice have sides numbered 1 through 8. Each side has the same probability (chance) of landing face up. The probability that the product of the two numbers on the sides that land face-up exceeds 36 is

(A) $\frac{5}{32}$ **(B)** $\frac{11}{64}$ **(C)** $\frac{3}{16}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{1}{2}$

21. Each corner cube is removed from this 3 cm x 3 cm x 3 cm cube. The surface area of the remaining figure is



(A) 19 sq.cm **(B)** 24 sq.cm **(C)** 30 sq.cm **(D)** 54 sq.cm **(E)** 72 sq.cm

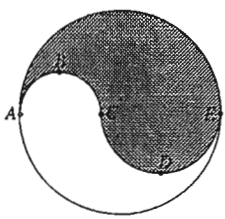
22. A two-inch cube (2x2x2) of silver weighs 3 pounds and is worth \$200. How much is a three-inch cube of silver worth?

(A) \$300 **(B)** \$375 **(C)** \$450 **(D)** \$560 **(E)** \$675

23. There are positive integers that have these properties:
    I. the sum of the squares of their digits is 50, and
    II. each digit is larger than the one to its left.
    The product of the digits of the largest integer with both properties is

(A) 7 **(B)** 25 **(C)** 36 **(D)** 48 **(E)** 60

24. Diameter ACE is divided at C in the ratio 2:3. The two semicircles, ABC and CDE, divide the circular region into an upper (shaded) region and a lower region. The ratio of the area of the upper region to that of the lower region is



(A) 2:3 **(B)** 1:1 **(C)** 3:2 **(D)** 9:4 **(E)** 5:2

25. All of the even numbers from 2 to 98 inclusive, except those ending in 0, are multiplied together. What is the rightmost digit (the units digit) of the product?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

SOLUTIONS

Your School Examination Manager will be sent at least one copy of the 1997 AJHSME Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

WRITE TO US!

Correspondence about the problems and solutions for this AJHSME should be addressed to:

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1998 AHSME

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Dr. Walter E. Mientka, AMC Executive Director
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University of Nebraska-Lincoln
P.O. Box 81606
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1997

American Junior High School Mathematics Examination
(AJHSME)

DO NOT OPEN UNTIL THURSDAY, NOVEMBER 20, 1997

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2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AJHSME CERTIFICATION Form that all rules associated with the conduct of the examination were followed.

3. The Answer Forms must be mailed by First Class Mail to Dr. Mientka no late than 24 hours following the Examination.

4. THE AJHSME IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAMINATION MAY BE GIVEN DURING THE REGULAR MATHEMATICS CLASS PERIOD OF THE STUDENTS IF IT IS NOT POSSIBLE TO ADMINISTER THE EXAMINATION TO ALL STUDENTS DURING ONE 40 MINUTE PERIOD.

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