AMERICAN MATHEMATICS COMPETITIONS

12th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 21, 1996

Sponsored by

Mathematical Association of America
Society of Actuaries Mu Alpha Theta
National Council of Teachers of Mathematics
Casualty Actuarial Society American Statistical Association
American Mathematical Association of Two-Year Colleges
American Mathematical Society
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INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AJHSME ANSWER FORM with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. No aids are permitted other than scratch paper, graph paper, ruler, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working the problems. You will have **40 MINUTES** working time for the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. How many positive factors of 36 are also multiples of 4?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

2. José, Thuy, and Kareem each start with the number 10. José subtracts 1 from the number 10, doubles his answer, and then adds 2. Thuy doubles the number 10, subtracts 1 from her answer, and then adds 2. Kareem subtracts 1 from the number 10, adds 2 to his answer, and then doubles the result. Who gets the largest final answer?

   (A) José **(B)** Thuy **(C)** Kareem

   (D) José and Thuy **(E)** Thuy and Kareem

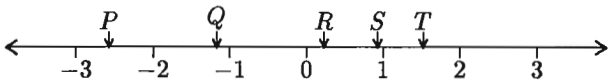
3. The 64 whole numbers from 1 through 64 are written, one per square, on a checkerboard (an 8 by 8 array of 64 squares). The first 8 numbers are written in order across the first row, the next 8 across the second row, and so on. After all 64 numbers are written, the sum of the numbers in the four corners will be

   (A) 130 **(B)** 131 **(C)** 132 **(D)** 133 **(E)** 134

4. $\dfrac{2+4+6+\cdots+34}{3+6+9+\cdots+51} =$

   (A) $\frac{1}{3}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{17}{3}$ **(E)** $\frac{34}{3}$

5. The letters P, Q, R, S, and T represent numbers located on the number line as shown.



   Which of the following expressions represents a negative number?

   (A) $P-Q$ **(B)** $P\cdot Q$ **(C)** $\dfrac{S}{Q}\cdot P$ **(D)** $\dfrac{R}{P\cdot Q}$ **(E)** $\dfrac{S+T}{R}$

6. What is the smallest result that can be obtained by the following process?
   - Choose three different numbers from the set $\{3,5,7,11,13,17\}$.
   - Add two of these numbers.
   - Multiply their sum by the third number.

   (A) 15 **(B)** 30 **(C)** 36 **(D)** 50 **(E)** 56

7. Brent has goldfish that quadruple (become four times as many) every month, and Gretel has goldfish that double every month. If Brent has 4 goldfish at the same time that Gretel has 128 goldfish, then in how many months from that time will they have the same number of goldfish?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

8. Points A and B are 10 units apart. Points B and C are 4 units apart. Points C and D are 3 units apart. If A and D are as close as possible, then the number of units between them is

   (A) 0 **(B)** 3 **(C)** 9 **(D)** 11 **(E)** 17

9. If 5 times a number is 2, then 100 times the reciprocal of the number is

   (A) 2.5 **(B)** 40 **(C)** 50 **(D)** 250 **(E)** 500

10. When Walter drove up to the gasoline pump, he noticed that his gasoline tank was 1/8 full. He purchased 7.5 gallons of gasoline for \$10. With this additional gasoline, his gasoline tank was then 5/8 full. The number of gallons of gasoline his tank holds when it is full is

    (A) 8.75 **(B)** 10 **(C)** 11.5 **(D)** 15 **(E)** 22.5

11. Let x be the number

$$0.\underbrace{0000\ldots0000}_{\text{1996 zeros}}1,$$

    where there are 1996 zeros after the decimal point. Which of the following expressions represents the largest number?

    (A) $3+x$ **(B)** $3-x$ **(C)** $3\cdot x$ **(D)** $3/x$ **(E)** $x/3$

12. What number should be removed from the list

$$1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11$$

    so that the average of the remaining numbers is 6.1?

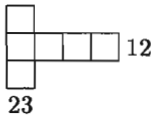
    (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

13. In the fall of 1996, a total of 800 students participated in an annual school clean-up day. The organizers of the event expect that in each of the years 1997, 1998, and 1999, participation will increase by 50% over the previous year. The number of participants the organizers expect in the fall of 1999 is

    (A) 1200 **(B)** 1500 **(C)** 2000 **(D)** 2400 **(E)** 2700

14. Six different digits from the set

$$\{1,2,3,4,5,6,7,8,9\}$$



are placed in the squares in the figure shown so that the sum of the entries in the vertical column is 23 and the sum of the entries in the horizontal row is 12. The sum of the six digits used is

(A) 27 **(B)** 29 **(C)** 31 **(D)** 33 **(E)** 35

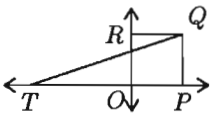
15. The remainder when the product $1492 \cdot 1776 \cdot 1812 \cdot 1996$ is divided by 5 is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

16. $1-2-3+4+5-6-7+8+9-10-11+12+13-\cdots$
$\cdots+1992+1993-1994-1995+1996=$

(A) -998 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 998

17. Figure $OPQR$ is a square. Point O is the origin, and point Q has coordinates $(2,2)$. What are the coordinates for T so that the area of triangle PQT equals the area of square $OPQR$?



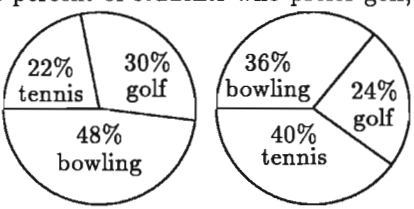
NOT TO SCALE

(A) $(-6,0)$ **(B)** $(-4,0)$ **(C)** $(-2,0)$
(D) $(2,0)$ **(E)** $(4,0)$

18. Ana's monthly salary was \$2000 in May. In June she received a 20% raise. In July she received a 20% pay cut. After the two changes in June and July, Ana's monthly salary was

(A) \$1920 **(B)** \$1980 **(C)** \$2000 **(D)** \$2020 **(E)** \$2040

19. The pie charts at the right indicate the percent of students who prefer golf, bowling, or tennis at East Junior High School and West Middle School. The total number of students at East is 2000 and at West, 2500. In the two schools combined, the percent of students who prefer tennis is



East JHS
2000 students

West MS
2500 students

(A) 30% **(B)** 31%
(C) 32% **(D)** 33% **(E)** 34%

20. Suppose there is a special key on a calculator that replaces the number x currently displayed with the number given by the formula $1/(1-x)$. For example, if the calculator is displaying 2 and the special key is pressed, then the calculator will display -1 since $1/(1-2) = -1$. Now suppose that the calculator is displaying 5. After the special key is pressed 100 times in a row, the calculator will display

(A) -0.25 **(B)** 0 **(C)** 0.8 **(D)** 1.25 **(E)** 5

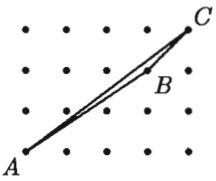
21. How many subsets containing three different numbers can be selected from the set

$$\{89, 95, 99, 132, 166, 173\}$$

so that the sum of the three numbers is even?

(A) 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 24

22. The horizontal and vertical distances between adjacent points equal 1 unit. The area of triangle ABC is

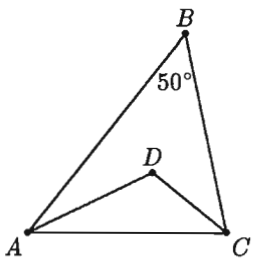


(A) 1/4 **(B)** 1/2 **(C)** 3/4
(D) 1 **(E)** 5/4

23. The manager of a company planned to distribute a \$50 bonus to each employee from the company fund, but the fund contained \$5 less than what was needed. Instead the manager gave each employee a \$45 bonus and kept the remaining \$95 in the company fund. The amount of money in the company fund before any bonuses were paid was

(A) \$945 **(B)** \$950 **(C)** \$955 **(D)** \$990 **(E)** \$995

24. The measure of angle ABC is $50°$, $\overline{AD}$ bisects angle BAC, and $\overline{DC}$ bisects angle BCA. The measure of angle ADC is



(A) 90° **(B)** 100° **(C)** 115°
(D) 122.5° **(E)** 125°

25. A point is chosen at random from within a circular region. What is the probability that the point is closer to the center of the region than it is to the boundary of the region?

(A) 1/4 **(B)** 1/3 **(C)** 1/2 **(D)** 2/3 **(E)** 3/4

SOLUTIONS

Your School Examination Manager will be sent at least one copy of the 1996 AJHSME Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

WRITE TO US!

Correspondence about the problems and solutions for this AJHSME should be addressed to:

Mr Bruce Brombacher, AJHSME Chairman
Jones Middle School
Upper Arlington, OH 43221

Comments about administrative arrangements should be addressed to:

Prof Walter E Mientka, AMC Executive Director
Department of Mathematics and Statistics, University of Nebraska
Lincoln, NE 68588-0658; Phone: 402-472-2257; Fax: 402-472-6087

1997 AHSME

The American High School Mathematics Examination [AHSME] is a 30-question, 90-minute, multiple choice examination. Schools with high-scoring students on the AJHSME will receive a 1997 AHSME Invitation Brochure containing information about the AHSME and the registration procedure. The best way to prepare for the AHSME is to study the exams from previous years. Orders for all publications listed below should be addressed to:

Dr. Walter E. Mientka, AMC Executive Director
American Mathematics Competitions
University of Nebraska-Lincoln
P.O. Box 81606
Lincoln, NE 68501-1606

PUBLICATIONS

MINIMUM ORDER: $5 (before handling fee), US FUNDS ONLY. Canada and US orders must be prepaid. Orders are mailed 4th class, unless you specify 1st class, in which case add 20% of the total order, with a minimum of $3 and a maximum of $15. **Please note that if the correct 1st class cost is not included, the order will be sent 4th class.** Make checks payable to the American Mathematics Competitions; or give Visa or MasterCard number, expiration date and cardholders home address.

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Examinations: Each price is for an examination **and** its solutions for one year. Specify the years you want and how many copies of each. All prices effective to September 1, 1997.

- **AJHSME** (Junior High Exam), 1985-1996, $1 per copy per year.
- **AHSME** (High School Exam) 1980-96, $1 per copy per year.
- **AJHSME Summary** of Results and Awards, 1985-96, $5 per copy per year.
- **AHSME Summary** of Results and Awards, 1980-96, $10 per copy per year.

Books (Exams and Solutions):

- Problem Book I, AHSMEs 1950-60, $8.00
- Problem Book II, AHSMEs 1961-65, $8.00
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- Problem Book IV, AHSMEs 1973-82, $13.50
- USA Mathematical Olympiad Book 1972-86, $16.00
- International Mathematical Olympiad Book I, 1959-77, $14.00
- International Mathematical Olympiad Book II, 1978-85, $11.50

1996

American Junior High School Mathematics Examination

(AJHSME)

DO NOT OPEN UNTIL

THURSDAY, NOVEMBER 21, 1996

*****Administration On An Earlier Date Will Disqualify Your School's Results*****

1. All information (Rules and Instructions) needed to administer the AJHSME is contained in the AJHSME TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE NOVEMBER 21.** Nothing is needed from inside this package until November 21.

2. Your PRINCIPAL or VICE PRINCIPAL must verify on the AJHSME CERTIFICATION Form that all rules associated with the conduct of the examination were followed.

3. The Answer Forms must be mailed by First Class Mail to Dr. Mientka no later than 24 hours following the Examination.

4. THE AJHSME IS TO BE ADMINISTERED DURING A CONVENIENT 40-MINUTE PERIOD. THE EXAMINATION MAY BE GIVEN DURING THE REGULAR MATHEMATICS CLASS PERIOD OF THE STUDENTS IF IT IS NOT POSSIBLE TO ADMINISTER THE EXAMINATION TO ALL STUDENTS DURING ONE 40-MINUTE PERIOD.