AMERICAN MATHEMATICS COMPETITIONS

AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

11th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 16, 1995

Sponsored by

Mathematical Association of America
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This Solutions Pamphlet gives at least one solution for each problem on this year's Examination and shows that all the problems can be solved using material normally associated with the mathematics curriculum for the eighth grade or lower. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

We hope that teachers will share these solutions with their students. *However, photocopying this material is a violation of the copyright.*

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1. **(D)**. The total value of the coins is

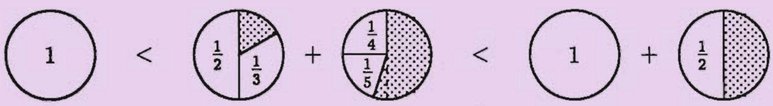
$$\$0.01 + \$0.05 + \$0.10 + \$0.25 = \$0.41,$$

which is $0.41/1.00$ or $41/100 = 41\%$ of a dollar.

2. **(C)** Zack must be $15 + 3 = 18$ years old, so Jose must be $18 - 4 = 14$ years old.

3. **(E)** Since $\frac{3}{4} \div \frac{3}{5} = \frac{3}{4} \times \frac{5}{3} = \frac{5}{4}$, multiplying by $\frac{5}{4}$ has the same effect.

4. **(C)** Ben adds 1 to 6 to get 7, and then doubles 7 to get 14. Sue subtracts 1 from 14 to get 13, and then doubles 13 to get 26.

5. **(C)** 

The sum of the fractions adds between 1 and $1\frac{1}{2}$ to the sum of the whole numbers, which is $2 + 3 + 4 + 5 = 14$. Thus the overall sum is between 15 and $15\frac{1}{2}$, so the answer is 16.

OR

Since

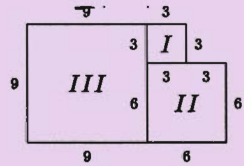
$$1 < \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{5} < \frac{\mathbf{1}}{\mathbf{2}} + \frac{\mathbf{1}}{\mathbf{3}} + \frac{\mathbf{1}}{\mathbf{4}} + \frac{\mathbf{1}}{\mathbf{5}} < \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2,$$

the sum of the fractions adds between 1 and 2 to the sum of the whole numbers, which is $2 + 3 + 4 + 5 = 14$. Thus the overall sum is between 15 and 16, so the answer is 16.

OR

Approximate the fractions as decimals and add $2.5 + 3.33 + 4.25 + 5.2$ which yields 15.28. Thus the answer is 16.

6. **(C)** Since the perimeter of I is 12, its side is 3. Similarly, the side of II is 6. Hence the side of III is $3 + 6 = 9$. Thus the perimeter of III is $4 \times 9 = 36$.



Query. Is it a coincidence that the perimeter of III equals the sum of the perimeters of I and II?

7. **(B)** $\frac{1}{2}+\frac{1}{4}+\frac{1}{10}=\frac{10}{20}+\frac{5}{20}+\frac{2}{20}=\frac{17}{20}$. The rest, $\frac{20}{20}-\frac{17}{20}=\frac{3}{20}$, walk home.

OR

Since $\frac{1}{2}$ or 50% of the students use the bus,

$\frac{1}{4}$ or 25% of the students use an auto, and

$\frac{1}{10}$ or 10% of the students use a bicycle,

it follows that $100\%-(50\%+25\%+10\%)=15\%$ or $\frac{3}{20}$ of the students walk home.

OR

Assume there are 100 students. This yields

$\frac{1}{2}$		$\frac{1}{4}$		$\frac{1}{10}$				
50	+	25	+	10	+	?	=	100 total.
bus		automobile		bicycle		walk		

Thus 15 walk home, which gives $\frac{15}{100}=\frac{3}{20}$.

8. **(D)** $\$1.00=\$1.00\times\frac{3000\text{ lire}}{\$1.60}=1875$ lire.

OR

$\frac{\$1.00}{\$1.60}=\frac{5}{8}$, so \$1.00 is $\frac{5}{8}$ of \$1.60. Thus, the number of lire is $\frac{5}{8}$ of 3000 or 1875 lire.

OR

Use the proportion $\frac{3000\text{ lire}}{160\text{ cents}}=\frac{x\text{ lire}}{100\text{ cents}}$. Solving for x gives 1875 lire.

9. **(C)** Since the length of $\overline{BC}$ is the same as the diameter of the circle with center Q, it follows that $BC = 4$. Since the circles with centers P and R are tangent to the parallel sides $\overline{AB}$ and $\overline{DC}$, the diameters of these circles are also 4. The sum of the diameters of the circles with centers P and R gives the length of $\overline{AB}$, so $AB = 4 + 4 = 8$. Hence the area of the rectangle is $8 \times 4 = 32$.

OR

The radius of the circle with center Q is $4/2 = 2$, so $PQ = RQ = 2$. But $\overline{PQ}$ and $\overline{RQ}$ are also radii of the circles with centers P and R, respectively, so all three circles have radius 2. Hence $AB = 8$ and $BC = 4$, so the area of the rectangle is $8 \times 4 = 32$.

10. **(A)** For the jacket, 40% of \$80 is a savings of \$32. For the shirt, 55% of \$40 is a savings of \$22. The total savings is \$32 + \$22 = \$54. The total of the original prices is \$120. Thus, \$54/\$120 = 0.45 = 45%.

11. **(D)** Jane covers twice as much distance as Hector in the same time. So Jane goes to A then B, as Hector goes to E. Jane continues to C then D as Hector goes to D.

OR

Jane covers twice as much distance as Hector in the same time. When they meet, they will have covered 18 blocks. Since $12 + 6 = 18$ and 12 is twice 6, they must meet 6 blocks counterclockwise from the start, and this is point D.

12. **(E)** The last two digits of 1994 can only be factored as $94 = 2 \times 47$. All the other choices have at least one date that makes them lucky:
(A) 9/10/90 **(B)** 7/13/91 **(C)** 4/23/92 **(D)** 3/31/93

13. **(E)** In $\triangle BDE$, $\angle BED + \angle BDE + \angle B = 180°$. Since $\angle BED = \angle BDE$ and $\angle B = 90°$, it follows that $\angle BED = \angle BDE = 45°$. In $\triangle AEF$, $\angle A + \angle AEF + \angle AFE = 180°$. Since $\angle A = 90°$ and $\angle AEF = 40°$, it follows that $\angle AFE = 50°$. Consequently $\angle BFG = 50°$ in $\triangle BFG$ and, since $\angle B = 90°$, it follows that $\angle BGF = 40°$. Consequently $\angle CGD = 40°$ in $\triangle CDG$, and since $\angle C = 90°$, it follows that $\angle CDG = 50°$. Thus $\angle CDE = 50° + 45° = 95°$.

OR

As in the first solution, $\angle BED = \angle BDE = 45°$. Then $\angle AED = 40° + 45° = 85°$. Since the four angles of a quadrilateral sum to 360°, we have $\angle A + \angle C + \angle AED + \angle CDE = 360°$. Thus $\angle CDE = 360° - 90° - 90° - 85° = 95°$.

14. **(B)** The total number of games for the season is $50 + 40 = 90$ games. Since 70% of 90 games is 63, it follows that $63 - 40 = 23$ more wins are needed.

15. **(B)** Since $4/37 = 0.108108\ldots$, it follows that the 3^{rd}, 6^{th}, 9^{th} $\ldots$, 99^{th} digits are 8. Thus the 100^{th} digit must be 1.

16. **(C)** Tabulate the data:

Allen School	: 7 students for 3 days	=	21 worker days
Balboa School	: 4 students for 5 days	=	20 worker days
Carver School	: 5 students for 9 days	=	45 worker days
	Total :		86 worker days

Hence, $\$774 \div 86 = \$9$ per worker day. Thus the students from Balboa School earned \$9 per worker day for 20 worker days, for a total of \$180.

17. **(D)** In Annville 11% of 100, or 11 students are in the 6^{th} grade. In Cleona 17% of 200, or 34 students are in the 6^{th} grade. Thus in the two schools combined, 45 out of 300 students are in the 6^{th} grade, so $45/300 = 0.15 = 15\%$ of all the students are in the 6^{th} grade.

OR

Since 1/3 of the students are at Annville and 2/3 are at Cleona, using a weighted average yields $\frac{1}{3}(0.11) + \frac{2}{3}(0.17) = 0.15$ or 15%.

18. **(C)** The four L-shaped regions account for $4 \times \left(\frac{3}{16}\right) = \frac{12}{16} = \frac{3}{4}$ of the total area. That leaves $\frac{1}{4}$ of the total area for the center square, which yields $\left(\frac{1}{4}\right)(100 \times 100) = 2500$ square inches. Thus the length of the side of the center square is $\sqrt{2500} = 50$ inches.

OR

As in the first solution, the center square is 1/4 of the total area. Dividing the large square into four equal parts as shown yields a square that is 50 by 50 inches.

19. **(D)** Putting the number of children in each family in order from least to greatest yields

$$1,1,2,3,3,4,\ \boxed{4},\ 5,5,5,5,5,5.$$

The median is the middle, or 7^{th} value, which is 4.

OR

The graph shows 2 families with 1 child, 1 with 2, 2 with 3, 2 with 4 and 6 with 5. Since there are $2+1+2+2+6=13$ families under consideration, the middle entry is the 7^{th} entry, which is a family with 4 children.

20. **(B)** There are $6 \times 6 = 36$ possible outcomes of rolling the dice. Since Diana and Apollo roll the same number in 6 of these, there are 30 in which the numbers on the two dice are different. By symmetry, Diana's number is larger than Apollo's number in exactly half of these. Thus the requested probability is $\dfrac{15}{36} = \dfrac{5}{12}$.

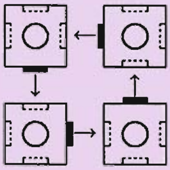
OR

Let (d, a) represent *"Diana rolled d and Apollo rolled a."* List the 36 outcomes and mark those where $d > a$.

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	**(3,2)**	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	**(4,2)**	**(4,3)**	(4,4)	(4,5)	(4,6)
(5,1)	**(5,2)**	**(5,3)**	**(5,4)**	(5,5)	(5,6)
(6,1)	**(6,2)**	**(6,3)**	**(6,4)**	**(6,5)**	(6,6)

Since there are 15 marked pairs, the probability that Diana rolls a larger number than Apollo is $15/36 = 5/12$.

21. **(B)** Using one, two or three cubes always leaves one protuding snap showing. The smallest number of cubes is four, arranged as shown (viewed from above).



22. **(A)** The prime factorization of 6545 is $5 \times 7 \times 11 \times 17$. Since the product of any three of these primes is a three-digit number and since 7×17 and 11×17 are both three-digit numbers, it follows that the only pair of two-digit numbers with product 6545 is $5 \times 17 = 85$ and $7 \times 11 = 77$. Thus the answer is $85 + 77 = 162$.

23. **(B)** There are 5 odd and 5 even digits that can be used for the two leftmost digits in the number. Once an odd and even digit have been selected and since all four digits are different, there are 8 choices remaining for the third digit, and then 7 choices for the fourth digit. Thus there are $5 \times 5 \times 8 \times 7 = 1400$ such whole numbers.

Query. How many four-digit whole numbers have an even leftmost digit, an odd second digit, and four different digits? (It is NOT 1400.)

24. **(C)** Since opposite sides of a parallelogram are equal, $AB = 12$. Then $AE = 12 - 4 = 8$. Using the Pythagorean Theorem gives $AD = \sqrt{8^2 + 6^2} = 10$, and then $BC = 10$ also. The area of a parallelogram is base $\times$ altitude. Using base $AB = 12$ and altitude $DE = 6$ gives an area of $12 \times 6 = 72$. Using base $BC = 10$ and altitude $\overline{DF}$ must also give an area of 72. Thus $DF = 72/10 = 7.2$.

25. **(D)** A Houston-bound bus leaving Dallas at 6:00 p.m., for example, will arrive in Houston at 11:00 p.m., having passed buses that left Houston at 1:30 p.m., 2:30 p.m., 3:30 p.m., ..., 10:30 p.m. That is 10 buses.