AMERICAN MATHEMATICS COMPETITIONS

10th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 17, 1994

Sponsored by

Mathematical Association of America
Society of Actuaries Mu Alpha Theta
National Council of Teachers of Mathematics
Casualty Actuarial Society American Statistical Association
American Mathematical Association of Two-Year Colleges
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INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AJHSME ANSWER FORM with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer sheet will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. No aids other than calculators, scratch paper, graph paper, rulers and erasers are permitted. No problems on the test will *require* the use of a calculator.
6. Unless specified otherwise, figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working the problems. You will have **40 MINUTES** working time for the test.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. Which of the following is the largest?

   (A) $\frac{1}{3}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{5}{12}$ **(E)** $\frac{7}{24}$

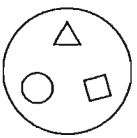
2. $\frac{1}{10}+\frac{2}{10}+\frac{3}{10}+\frac{4}{10}+\frac{5}{10}+\frac{6}{10}+\frac{7}{10}+\frac{8}{10}+\frac{9}{10}+\frac{55}{10}=$

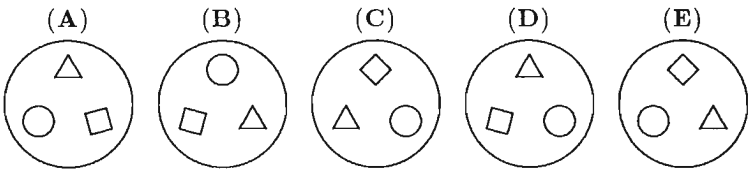
   (A) $4\frac{1}{2}$ **(B)** 6.4 **(C)** 9 **(D)** 10 **(E)** 11

3. Each day Maria must work 8 hours. This does not include the 45 minutes she takes for lunch. If she begins working at 7:25 A.M. and takes her lunch break at noon, then her working day will end at

   (A) 3:40 P.M. **(B)** 3:55 P.M. **(C)** 4:10 P.M.
   (D) 4:25 P.M. **(E)** 4:40 P.M.

4. Which of the following represents the result when the figure shown at the right is rotated clockwise 120° about its center?





5. Given that 1 mile = 8 furlongs and 1 furlong = 40 rods, the number of rods in one mile is

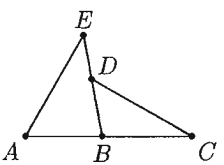
   (A) 5 **(B)** 320 **(C)** 660 **(D)** 1760 **(E)** 5280

6. The unit's digit (one's digit) of the product of any six consecutive positive whole numbers is

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

7. If $\angle A = 60°$, $\angle E = 40°$ and $\angle C = 30°$, then $\angle BDC =$

   (A) 40° **(B)** 50° **(C)** 60°
   (D) 70° **(E)** 80°



8. For how many three-digit whole numbers does the sum of the digits equal 25?

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

9. A shopper buys a \$100 coat on sale for 20% off. An additional \$5 is taken off the sale price by using a discount coupon. A sales tax of 8% is paid on the final selling price. The total amount the shopper pays for the coat is

(A) \$81.00 (B) \$81.40 (C) \$82.00 (D) \$82.08 (E) \$82.40

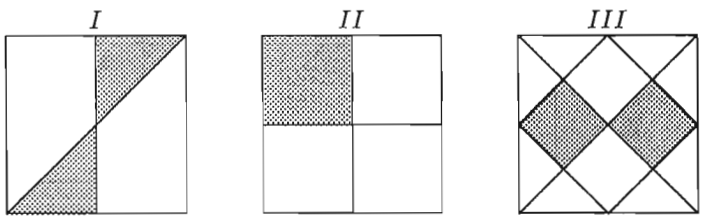
10. For how many positive integer values of N $(N > 0)$ is the expression $\frac{36}{N+2}$ an integer?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 12

11. Last summer 100 students attended basketball camp. Of those attending, 52 were boys and 48 were girls. Also, 40 students were from Jones Middle School and 60 were from Clay Middle School. Twenty of the girls were from Jones Middle School. How many of the boys were from Clay Middle School?

(A) 20 (B) 32 (C) 40 (D) 48 (E) 52

12. Each of the three large squares shown below is the same size. Segments that intersect the sides of the squares intersect at the midpoints of the sides. How do the shaded areas of these squares compare?



(A) The shaded areas in all three are equal.

(B) Only the shaded areas of I and II are equal.

(C) Only the shaded areas of I and III are equal.

(D) Only the shaded areas of II and III are equal.

(E) The shaded areas of I, II and III are all different.

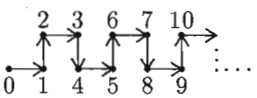
13. The number halfway between $\frac{1}{6}$ and $\frac{1}{4}$ is

(A) $\frac{1}{10}$ (B) $\frac{1}{5}$ (C) $\frac{5}{24}$ (D) $\frac{7}{24}$ (E) $\frac{5}{12}$

14. Two children at a time can play pairball. For 90 minutes, with only two children playing at one time, five children take turns so that each one plays the same amount of time. The number of minutes each child plays is

(A) 9 (B) 10 (C) 18 (D) 20 (E) 36

15. If this path is to continue in the same pattern:



then which sequence of arrows goes from point 425 to point 427?

(A)  (B)  (C)  (D)  (E) 

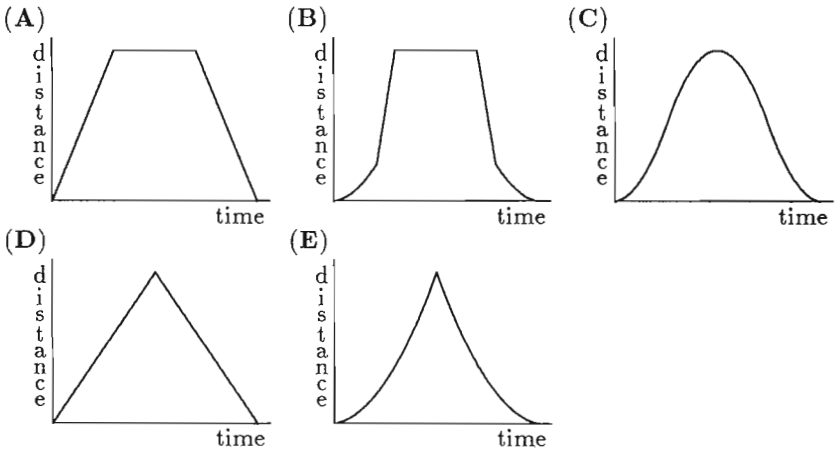
16. The perimeter of one square is 3 times the perimeter of another square. The area of the larger square is how many times the area of the smaller square?

(A) 2 (B) 3 (C) 4 (D) 6 (E) 9

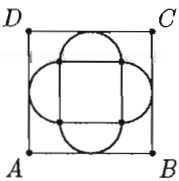
17. Pauline Bunyan can shovel snow at the rate of 20 cubic yards for the first hour, 19 cubic yards for the second, 18 for the third, etc., always shoveling one cubic yard less per hour than the previous hour. If her driveway is 4 yards wide, 10 yards long, and covered with snow 3 yards deep, then the number of hours it will take her to shovel it clean is closest to

(A) 4 (B) 5 (C) 6 (D) 7 (E) 12

18. Mike leaves home and drives slowly east through city traffic. When he reaches the highway he drives east more rapidly until he reaches the shopping mall where he stops. He shops at the mall for an hour. Mike returns home by the same route as he came, driving west rapidly along the highway and then slowly through city traffic. Each graph shows the distance from home on the vertical axis versus the time elapsed since leaving home on the horizontal axis. Which graph is the best representation of Mike's trip?



19. Around the outside of a 4 by 4 square, construct four semicircles (as shown in the figure) with the four sides of the square as their diameters. Another square, $ABCD$, has its sides parallel to the corresponding sides of the original square, and each side of $ABCD$ is tangent to one of the semicircles. The area of the square $ABCD$ is



(A) 16 **(B)** 32 **(C)** 36 **(D)** 48 **(E)** 64

20. Let W, X, Y and Z be four different digits selected from the set

$$\{1,2,3,4,5,6,7,8,9\}.$$

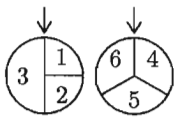
If the sum $\frac{W}{X}+\frac{Y}{Z}$ is to be as small as possible, then $\frac{W}{X}+\frac{Y}{Z}$ must equal

(A) $\frac{2}{17}$ **(B)** $\frac{3}{17}$ **(C)** $\frac{17}{72}$ **(D)** $\frac{25}{72}$ **(E)** $\frac{13}{36}$

21. A gumball machine contains 9 red, 7 white, and 8 blue gumballs. The least number of gumballs a person must buy to be *sure* of getting four gumballs of the same color is

(A) 8 (B) 9 (C) 10 (D) 12 (E) 18

22. The two wheels shown at the right are spun and the two resulting numbers are added. The probability that the sum of the two numbers is even is



(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{5}{12}$ (E) $\frac{4}{9}$

23. If X, Y and Z are different digits, then the largest possible 3-digit sum for

$$\begin{array}{r} XXX \\ YX \\ +\quad X \\ \hline \end{array}$$

has the form

(A) XXY (B) XYZ (C) YYX (D) YYZ (E) ZZY

24. A 2 by 2 square is divided into four 1 by 1 squares. Each of the small squares is to be painted either green or red. In how many different ways can the painting be accomplished so that no green square shares its top or right side with any red square? There may be as few as zero or as many as four small green squares.

(A) 4 (B) 6 (C) 7 (D) 8 (E) 16

25. Find the sum of the digits in the answer to

$$\underbrace{9999\cdots99}_{\text{94 nines}} \times \underbrace{4444\cdots44}_{\text{94 fours}}$$

where a string of 94 nines is multiplied by a string of 94 fours.

(A) 846 (B) 855 (C) 945 (D) 954 (E) 1072

SOLUTIONS

Your School Examination Manager will be sent at least one copy of the 1994 AJHSME Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

WRITE TO US!

Correspondence about the problems and solutions for this AJHSME should be addressed to:

Mr Bruce Brombacher, AJHSME Chairman
Jones Middle School
Upper Arlington, OH 43221

Comments about administrative arrangements and orders for any publications listed below should be addressed to:

Prof Walter E Mientka, AMC Executive Director
Department of Mathematics and Statistics, University of Nebraska
Lincoln, NE 68588-0658; Phone: 402-472-2257; Fax: 402-472-6087

1995 AHSME

The American High School Mathematics Examination [AHSME] is a 30-question, 90-minute, multiple choice examination. Schools with high-scoring students on the AJHSME will receive a 1995 AHSME Invitation Brochure containing information about the AHSME and registration procedure. The best way to prepare for the AHSME is to study the exams from previous years. The procedure used to purchase these publications is indicated below.

PUBLICATIONS

MINIMUM ORDER: $5 (before handling fee), US FUNDS ONLY. Canada and US orders must be prepaid. Orders are mailed 4th class, unless you specify 1st class, in which case add 20% of the total order, with a minimum of $3 and a maximum of $15. Make checks payable to the American Mathematics Competitions; or give Visa or MasterCard number and expiration date.

FOREIGN ORDERS: Do **NOT** prepay; an invoice will be sent.

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Examinations: Each price is for an examination **and** its solutions for one year. Specify the years you want and how many copies of each. All prices effective to July 1, 1995.

- **AJHSME** (Junior High Exam), 1985-1994, $1 per copy per year.
- **AHSME** (High School Exam) 1980-94, $1 per copy per year.
- **AJHSME Summary** of Results and Awards, 1985-93, $4 per copy per year.
- **AHSME Summary** of Results and Awards, 1980-94, $5 per copy per year.

Books (Exams and Solutions):

- Problem Book I, AHSMEs 1950-60, $9.50
- Problem Book II, AHSMEs 1961-65, $9.50
- Problem Book III, AHSMEs 1966-72, $11.50
- Problem Book IV, AHSMEs 1973-82, $11.50
- USA Mathematical Olympiad Book 1972-86, $14.50
- International Mathematical Olympiad Book I, 1959-77, $11.50
- International Mathematical Olympiad Book II, 1978-85, $11.50

1994

American Junior High School Mathematics Examination

(AJHSME)

DO NOT OPEN UNTIL THURSDAY, NOVEMBER 17, 1994

*****Administration On An Earlier Date Will Disqualify Your School's Results*****

1. All information (Rules and Instructions) needed to administer the AJHSME is contained in the AJHSME TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE NOVEMBER 17.** Nothing is needed from inside this package until November 17.

2. Your PRINCIPAL or VICE PRINCIPAL must verify on the AJHSME CERTIFICATION Form that all rules associated with the conduct of the examination were followed.

3. The Answer Forms must be mailed by First Class Mail to Dr. Mientka no later than 48 hours following the Examination.

4. THE AJHSME IS TO BE ADMINISTERED DURING A CONVENIENT 40-MINUTE PERIOD. THE EXAMINATION MAY BE GIVEN DURING THE REGULAR MATHEMATICS CLASS PERIOD OF THE STUDENTS IF IT IS NOT POSSIBLE TO ADMINISTER THE EXAMINATION TO ALL STUDENTS DURING ONE 40-MINUTE PERIOD.