AMERICAN MATHEMATICS COMPETITIONS

9th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 18, 1993

Sponsored by

Mathematical Association of America
Society of Actuaries Mu Alpha Theta
National Council of Teachers of Mathematics
Casualty Actuarial Society American Statistical Association
American Mathematical Association of Two-Year Colleges
American Mathematical Society

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AJHSME ANSWER FORM with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer sheet will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. No aids other than calculators, scratch paper, graph paper, rulers and erasers are permitted. No problems on the test will *require* the use of a calculator.
6. Unless specified otherwise, figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working the problems. You will have **40 MINUTES** working time for the test.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. Which pair of numbers does NOT have a product equal to 36?

   (A) $\{-4,-9\}$ **(B)** $\{-3,-12\}$ **(C)** $\left\{\frac{1}{2},-72\right\}$

   (D) $\{1,36\}$ **(E)** $\left\{\frac{3}{2},24\right\}$

2. When the fraction $\frac{49}{84}$ is expressed in simplest form, then the sum of the numerator and the denominator will be

   (A) 11 **(B)** 17 **(C)** 19 **(D)** 33 **(E)** 133

3. Which of the following numbers has the largest prime factor?

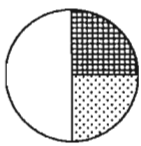
   (A) 39 **(B)** 51 **(C)** 77 **(D)** 91 **(E)** 121

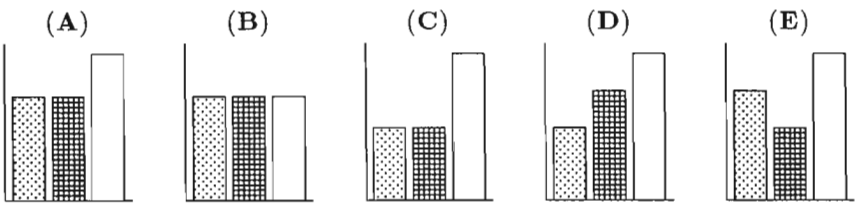
4. $1000 \times 1993 \times 0.1993 \times 10 =$

   (A) 1.993×10^3 **(B)** 1993.1993 **(C)** $(199.3)^2$

   (D) 1,993,001.993 **(E)** $(1993)^2$

5. Which one of the following bar graphs could represent the data from the circle graph?





6. A can of soup can feed 3 adults or 5 children. If there are 5 cans of soup and 15 children are fed, then how many adults would the remaining soup feed?

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 10

7. $3^3 + 3^3 + 3^3 =$

   (A) 3^4 **(B)** 9^3 **(C)** 3^9 **(D)** 27^3 **(E)** 3^{27}

8. To control her blood pressure, Jill's grandmother takes one half of a pill every other day. If one supply of medicine contains 60 pills, then the supply of medicine will last approximately

(A) 1 month **(B)** 4 months **(C)** 6 months
(D) 8 months **(E)** 1 year

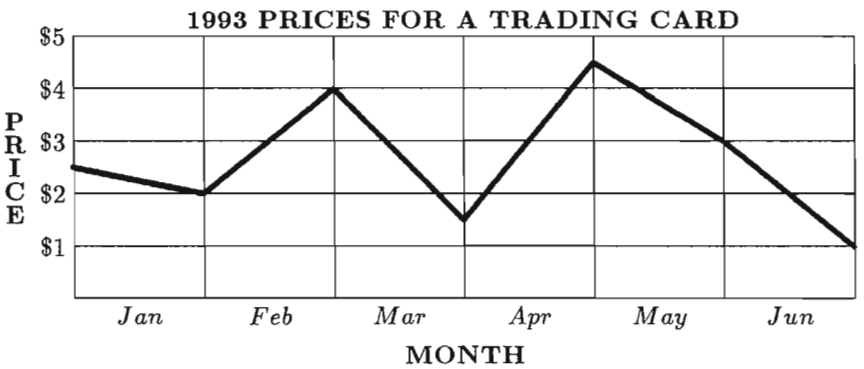
9. Consider the operation $*$ defined by the following table:

$*$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

For example, $3 * 2 = 1$. Then $(2 * 4) * (1 * 3) =$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

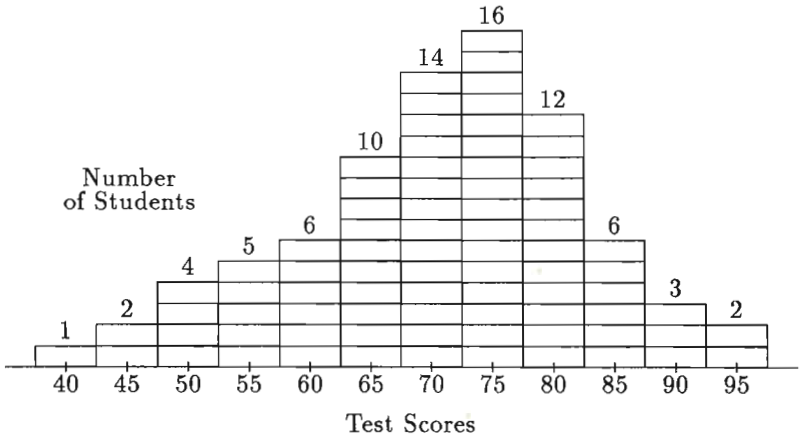
10. This line graph represents the price of a trading card during the first 6 months of 1993.



The greatest monthly drop in price occurred during

(A) January **(B)** March **(C)** April **(D)** May **(E)** June

11. Consider this histogram of the scores for 81 students taking a test:



The median is in the interval labeled

(A) 60 **(B)** 65 **(C)** 70 **(D)** 75 **(E)** 80

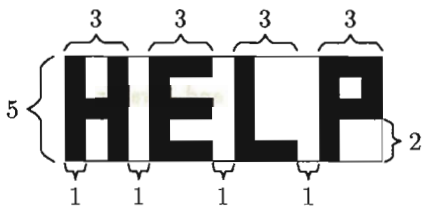
12. If each of the three operation signs, $+$, $-$, $\times$, is used exactly ONCE in one of the blanks in the expression

$$5 \_\_ 4 \_\_ 6 \_\_ 3$$

then the value of the result could equal

(A) 9 **(B)** 10 **(C)** 15 **(D)** 16 **(E)** 19

13. The word "**HELP**" in block letters is painted in black with strokes 1 unit wide on a 5 by 15 rectangular white sign with dimensions as shown. The area of the white portion of the sign, in square units, is



(A) 30 **(B)** 32 **(C)** 34 **(D)** 36 **(E)** 38

14. The nine squares in the table shown are to be filled so that every row and every column contains each of the numbers $1, 2, 3$. Then $A + B =$

1		
	2	A
		B

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

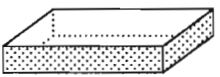
15. The arithmetic mean (average) of four numbers is 85. If the largest of these numbers is 97, then the mean of the remaining three numbers is

(A) 81.0 **(B)** 82.7 **(C)** 83.0 **(D)** 84.0 **(E)** 84.3

16. $\dfrac{1}{1+\dfrac{1}{2+\dfrac{1}{3}}} =$

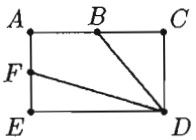
(A) $\frac{1}{6}$ **(B)** $\frac{3}{10}$ **(C)** $\frac{7}{10}$ **(D)** $\frac{5}{6}$ **(E)** $\frac{10}{3}$

17. Square corners, 5 units on a side, are removed from a 20 unit by 30 unit rectangular sheet of cardboard. The sides are then folded to form an open box. The surface area, in square units, of the interior of the box is



(A) 300 **(B)** 500 **(C)** 550 **(D)** 600 **(E)** 1000

18. The rectangle shown has length $AC = 32$, width $AE = 20$, and B and F are midpoints of $\overline{AC}$ and $\overline{AE}$, respectively. The area of the quadrilateral $ABDF$ is



(A) 320 **(B)** 325 **(C)** 330
(D) 335 **(E)** 340

19. $(1901 + 1902 + 1903 + \cdots + 1993) - (101 + 102 + 103 + \cdots + 193) =$

(A) 167,400 **(B)** 172,050 **(C)** 181,071 **(D)** 199,300 **(E)** 362,142

20. When $10^{93} - 93$ is expressed as a single whole number, the sum of the digits is

(A) 10 **(B)** 93 **(C)** 819 **(D)** 826 **(E)** 833

21. If the length of a rectangle is increased by 20% and its width is increased by 50%, then the area is increased by

(A) 10% **(B)** 30% **(C)** 70% **(D)** 80% **(E)** 100%

22. Pat Peano has plenty of 0's, 1's, 3's, 4's, 5's, 6's, 7's, 8's and 9's, but he has only twenty-two 2's. How far can he number the pages of his scrapbook with these digits?

(A) 22 **(B)** 99 **(C)** 112 **(D)** 119 **(E)** 199

23. Five runners, P, Q, R, S, T, have a race, and P beats Q, P beats R, Q beats S, and T finishes after P and before Q. Who could NOT have finished third in the race?

(A) P and Q **(B)** P and R **(C)** P and S
(D) P and T **(E)** P, S and T

24. What number is directly above 142 in this array of numbers?

$$\begin{array}{ccccc} & & 1 & & \\ & 2 & 3 & 4 & \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & \cdots & \end{array}$$

(A) 99 **(B)** 119 **(C)** 120 **(D)** 121 **(E)** 122

25. A checkerboard consists of one-inch squares. A square card, 1.5 inches on a side, is placed on the board so that it covers part or all of the area of each of n squares. The maximum possible value of n is

(A) 4 or 5 **(B)** 6 or 7 **(C)** 8 or 9 **(D)** 10 or 11 **(E)** 12 or more

SOLUTIONS

Your School Examination Manager has at least one copy of the 1993 AJHSME Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

WRITE TO US!

Correspondence about the problems and solutions for this AJHSME should be addressed to:

Mr Bruce Brombacher, AJHSME Chairman
Jones Middle School
Upper Arlington, OH 43221

Comments about administrative arrangements and orders for any publications listed below should be addressed to:

Prof Walter E Mientka, AMC Executive Director
Department of Mathematics and Statistics, University of Nebraska
Lincoln, NE 68588-0658; Phone: 402-472-2257; Fax: 402-472-6087

1994 AHSME

The American High School Mathematics Examination [AHSME] is a 30-question, 90-minute, multiple choice examination. Schools with high-scoring students on the AJHSME will receive a 1994 AHSME Invitation Brochure containing information about the AHSME and registration procedure. The best way to prepare for the AHSME is to study the exams from previous years. The procedure used to purchase these publications is indicated below.

PUBLICATIONS

MINIMUM ORDER: $5 (before handling fee), US FUNDS ONLY. Canada and US orders must be prepaid. Orders are mailed 4th class, unless you specify 1st class, in which case add $3 or 20% of total order, whichever is larger, with a maximum of $15. Make checks payable to the American Mathematics Competitions; or give Visa or MasterCard number and expiration date.

FOREIGN ORDERS: Do **NOT** prepay; an invoice will be sent.

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Examinations: Each price is for an examination **and** its solutions for one year. Specify the years you want and how many copies of each. All prices effective to July 1, 1994.

- **AJHSME** (Junior High Exam), 1985-1993, $1 per copy per year.
- **AHSME** (High School Exam) 1972-93, $1 per copy per year.
- **AJHSME Summary** of Results and Awards, 1985-92, $4 per copy per year.
- **AHSME Summary** of Results and Awards, 1980-93, $5 per copy per year.

Books (Exams and Solutions):

- Problem Book I, AHSMEs 1950-60, $9.50
- Problem Book II, AHSMEs 1961-65, $9.50
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- Problem Book IV, AHSMEs 1973-82, $11.50
- USA Mathematical Olympiad Book 1972-86, $14.50
- International Mathematical Olympiad Book I, 1959-77, $11.50
- International Mathematical Olympiad Book II, 1978-85, $11.50

1993

American Junior High School Mathematics Examination

(AJHSME)

DO NOT OPEN UNTIL THURSDAY, NOVEMBER 18, 1993

1. All information (Rules and Instructions) needed to administer the AJHSME is contained in the AJHSME TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE NOVEMBER 18.** Nothing is needed from inside this package until November 18.

2. Your PRINCIPAL or VICE PRINCIPAL must verify on the AJHSME CERTIFICATION Form that all rules associated with the conduct of the examination were followed.

3. The Answer Forms must be mailed by First Class Mail to Dr. Mientka no later than 48 hours following the Examination.

4. THE AJHSME IS TO BE ADMINISTERED DURING A CONVENIENT 40-MINUTE PERIOD. THE EXAMINATION MAY BE GIVEN DURING THE REGULAR MATHEMATICS CLASS PERIOD OF THE STUDENTS IF IT IS NOT POSSIBLE TO ADMINISTER THE EXAMINATION TO ALL STUDENTS DURING ONE 40-MINUTE PERIOD.