AMERICAN MATHEMATICS COMPETITIONS

AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

8th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 19, 1992

Sponsored by

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This Solutions Pamphlet gives at least one solution for each problem on this year's Examination and shows that all the problems can be solved using material normally associated with the mathematics curriculum for the eighth grade or lower. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

We hope that teachers will share these solutions with their students.

Questions and comments about the problems and solutions (but **not** requests for the Solutions Pamphlet) shoula be addressed to:

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1. **(B)** Group the numerator in pairs from the left, and group the denominator in pairs from the left:

$$\frac{\overbrace{10-9}^{1} + \overbrace{8-7}^{1} + \overbrace{6-5}^{1} + \overbrace{4-3}^{1} + \overbrace{2-1}^{1}}{\underbrace{1-2}_{-1} + \underbrace{3-4}_{-1} + \underbrace{5-6}_{-1} + \underbrace{7-8}_{-1} + 9}$$

Hence, the answer is $\dfrac{5(1)}{4(-1)+9} = \dfrac{5}{5} = 1.$

OR

Regroup the numerator and denominator into positive and negative terms,

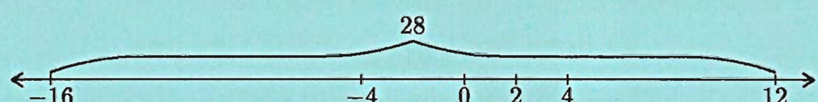
$$\frac{(10+8+6+4+2)-(9+7+5+3+1)}{(1+3+5+7+9)-(2+4+6+8)} = \frac{30-25}{25-20} = \frac{5}{5} = 1.$$

2. **(D)** $1\frac{1}{5} = \frac{6}{5} \neq \frac{5}{4}$.

3. **(D)** To obtain the largest difference, subtract the smallest number, -16, from the largest number, 12. Thus $12-(-16)=28$.

OR

Graphing the numbers on the number line, the difference is represented by the distance between two points. The largest difference would be represented by the longest distance between numbers, which is the distance between -16 and 12, a distance of 28.



4. **(E)** Judy had a total of 35 hits, of which $35-(1+1+5)=28$ were singles. Thus $\dfrac{28}{35} = \dfrac{4}{5}$ or 80% were singles.

OR

Out of 35 hits, $1+1+5=7$ were not singles, so $\dfrac{7}{35} = \dfrac{1}{5}$ or 20% were not singles. Thus $100\% - 20\% = 80\%$ were singles.

5. **(E)** The area of the circle is between $1/2$ and 1. To see this, draw squares around and inside the circle. The area of the large square is 1, the area of the small square is $1/2$, and the circle fits between the two squares. The area of the rectangle with the circle removed is therefore between 5 and 5.5, so the whole number closest to this area is 5.

OR

The area of the rectangle is $2 \times 3 = 6$, and the area of the circle with radius $\frac{1}{2}$ is $\pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}$, which is slightly larger than $\frac{3}{4}$. Thus the area of the resulting figure is slightly smaller than $6 - \frac{3}{4}$, so it is closest to 5.

6. **(D)** $(1+3-4)+(2+5-6) = 0+1 = 1$.

OR

Note that 

where $x = a+d$, $y = b+e$ and $z = c+f$; i.e., the sum of two 'triangular expressions' is the value of the 'triangular expression' obtained by summing the respective components. It follows that the required sum is $(1+2)+(3+5)-(4+6) = 3+8-10 = 1$.

7. **(A)** The only 3-digit whole numbers with a digit-sum of 26 are 899, 989 and 998. Of these, only 998 is even. Thus there is only one such number.

8. **(C)** Since he bought 1500 pencils at \$0.10 each, he paid $1500 \times \$0.10 = \$150$. To make \$100 profit he must take in $\$150 + \$100 = \$250$. Therefore, selling the pencils for \$0.25 each, he must sell $\$250 \div \$0.25 = 1000$ pencils.

9. **(B)** The ratio of males to the total population is 1 to 3. Thus, there are $1/3$ of 480, or 160 males in the town.

10. **(B)** The area of each of the small shaded triangles is $\frac{1}{2} \times 2 \times 2 = 2$. There are ten of these, so the shaded area is $2 \times 10 = 20$.

OR

The area of the large triangle is $\frac{1}{2} \times 8 \times 8 = 32$. Only 10 of the 16 small triangles are shaded. Thus the shaded area is $\frac{10}{16}$ of 32, or 20.

11. **(B)** The total frequency for all colors is $50 + 60 + 40 + 60 + 40 = 250$. The frequency for blue is 60. Thus the percent that preferred blue is $60/250$, or 24%.

12. **(C)** The total number of miles of wear is $30,000 \times 4 = 120,000$. Since this wear is shared equally by each of the 5 tires, each tire traveled $120,000 \div 5 = 24,000$ miles.

OR

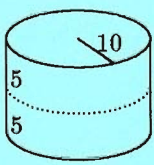
Since each of the tires was on for $\frac{4}{5}$ of the driving, it follows that each was used $\frac{4}{5} \times 30,000 = 24,000$ miles.

13. **(B)** If the mean is 90, then the sum of all five scores is $5 \times 90 = 450$. Since the median of the five scores is 91, at least one score must be 91 and two other scores must be greater than or equal to 91. Since 94 is the mode, there are two scores of 94. The sum of the remaining scores must equal $450 - (94 + 94 + 91) = 171$.

14. **(D)** Since 4 gallons is the difference between being $1/3$ full and $1/2$ full, it follows that 4 gallons is $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ of the capacity of the tank. Thus the capacity of the tank must be 24 gallons.

15. **(C)** The pattern repeats every 9 letters. Dividing 1992 by 9 yields a remainder of 3. Therefore, the 1992^{nd} letter corresponds to the third letter in the sequence, which is C.

16. **(B)** Cylinder **(B)** can be obtained by stacking one copy of the given cylinder on top of another. The formula for the volume of a cylinder with radius r and height h is $V = \pi r^2 h$. Use this to show that none of the other cylinders has twice the volume of the given cylinder:



Cylinder	Volume
Given :	$\pi \times 10^2 \times 5 = 500\pi$
(A) :	$\pi \times 20^2 \times 5 = 2000\pi$
(C) :	$\pi \times 5^2 \times 20 = 500\pi$
(D) :	$\pi \times 20^2 \times 10 = 4000\pi$

Note. If the radius remains the same and the height is doubled, then the volume will double, as in **(B)**. Doubling the radius while the height remains the same will multiply the volume by 4, as in **(A)**.

17. **(B)** For any triangle, the sum of the lengths of any two sides must be greater than the length of the third side. Thus $6.5 + s$ must be greater than 10. The smallest such whole number for s is 4.

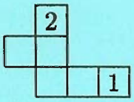
18. **(A)** During the 4 hours, the car traveled a total of $80 + 0 + 100 = 180$ miles for an average speed of $180/4 = 45$ miles per hour.

19. **(C)** There are 21 segments between the 5^{th} and 26^{th} exits. Using the minimum length of 5 miles, 20 segments would yield $20 \times 5 = 100$ miles. This leaves $118 - 100 = 18$ miles for the other segment.

OR

There are 21 segments between the 5^{th} and 26^{th} exits. If each segment were its minimal 5 miles length, then the total distance between the 5^{th} and 26^{th} exits would be 105 miles. Since $118 - 105 = 13$, all 13 additional miles could occur between one pair of consecutive exits. Such a pair would be $5 + 13 = 18$ miles apart.

20. **(D)** Any attempt to fold the squares would result in square 1 being superimposed on square 2. Have students cut and fold the other four patterns into cubes.



21. **(B)** Compute the ratios for each month:

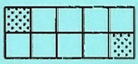
Month	Drums	Bugles	Diff.	Diff.:Lower	% exc.
Jan :	7	9	2	2:7	29%
Feb :	5	3	2	2:3	67%
Mar :	9	6	3	3:6	50%
Apr :	9	12	3	3:9	33%
May :	8	10	2	2:8	25%

Thus the percent is greatest in February.

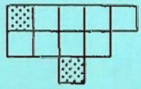
Note. Students can estimate the required ratio by visually comparing the difference between the columns to the shorter column.

22. **(C)** When a new tile is added to the original figure, it may have one or two sides in common with the given tiles, as shown. When a tile shares one side, the original perimeter is increased by 2. When a tile shares two sides, there is no change in the perimeter. By adding two tiles, the only possible changes to the perimeter are increases of 0, 2 or 4. Hence, the possible values of the perimeter are 14, 16 or 18.

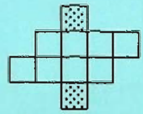
Note. Examples of the three possibilities are shown.



original = 14
new perimeter = 14



original = 14
new perimeter = 16



original = 14
new perimeter = 18

23. **(B)** Make a table and fill in the products greater than 10.

×	**1**	**2**	**3**	**4**	**5**	**6**
1 :	1	2	3	4	5	6
2 :	2	4	6	8	10	**12**
3 :	3	6	9	**12**	**15**	**18**
4 :	4	8	**12**	**16**	**20**	**24**
5 :	5	10	**15**	**20**	**25**	**30**
6 :	6	**12**	**18**	**24**	**30**	**36**

Since there are 17 such products out of a possible 36 products, the probability is 17/36.

24. **(A)** The four quarter-circles that lie inside the square have a total area equal to the area of one of the circles, 9π. The length of a side of the square is equal to two radii, 6, and thus the square has area 36. The difference is $36 - 9\pi < 36 - 9(3) = 9$, so it is closest to 7.7. (The area, to one decimal place, is 7.7.)

25. **(D)** After the first pouring, $\frac{1}{2}$ remains. After the second pouring $\frac{1}{2} \times \frac{2}{3}$ remains. After the third pouring $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$ remains. How many pourings until $\frac{1}{10}$ remains?

$$\frac{1}{\not 2} \times \frac{\not 2}{\not 3} \times \frac{\not 3}{\not 4} \times \frac{\not 4}{\not 5} \times \frac{\not 5}{\not 6} \times \frac{\not 6}{\not 7} \times \frac{\not 7}{\not 8} \times \frac{\not 8}{\not 9} \times \frac{\not 9}{10} = \frac{1}{10}$$

indicates 9 pourings.

OR

Make a table for the information:

Pouring	Amount Poured	Amount Remaining
1	$\frac{1}{2}$	$1 - \frac{1}{2} = \frac{1}{2}$
2	$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$	$\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$
3	$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$	$\frac{1}{3} - \frac{1}{12} = \frac{1}{4}$
4	$\frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$	$\frac{1}{4} - \frac{1}{20} = \frac{1}{5}$
$\vdots$	$\vdots$	$\vdots$
n	$\frac{1}{n+1} \times \frac{1}{n} = \frac{1}{n(n+1)}$	$\frac{1}{n} - \frac{1}{n(n+1)} = \frac{1}{n+1}$
$\vdots$	$\vdots$	$\vdots$
9	$\frac{1}{10} \times \frac{1}{9} = \frac{1}{90}$	$\frac{1}{9} - \frac{1}{90} = \frac{1}{10}$

Thus 1/10 remains after the 9^{th} pouring.