

AMERICAN MATHEMATICS COMPETITIONS

7th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 21, 1991

Sponsored by

Mathematical Association of America
Society of Actuaries Mu Alpha Theta
National Council of Teachers of Mathematics
Casualty Actuarial Society American Statistical Association
American Mathematical Association of Two-Year Colleges
American Mathematical Society

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. For each question, indicate your answer by marking the appropriate space on the answer form provided by your proctor.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. Use a #2 pencil since your answer form will be read by a marked-sense machine. Scratch paper, graph paper, rulers and erasers are permitted. Calculators may be used on the 1991 AJHSME. The nature of the questions has not changed from past tests. No problems on the test will require the use of a calculator, but you may have one available to use if you wish.
6. Unless specified otherwise, figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working the problems. You will have **40 MINUTES** working time for the test.


The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. $1,000,000,000,000 - 777,777,777,777 =$
 (A) 222,222,222,222 (B) 222,222,222,223 (C) 233,333,333,333
 (D) 322,222,222,223 (E) 333,333,333,333

2. $\frac{16 + 8}{4 - 2} =$
 (A) 4 (B) 8 (C) 12 (D) 16 (E) 20

3. Two hundred thousand times two hundred thousand equals
 (A) four hundred thousand
 (B) four million
 (C) forty thousand
 (D) four hundred million
 (E) forty billion

4. If $991 + 993 + 995 + 997 + 999 = 5000 - N$, then $N =$
 (A) 5 (B) 10 (C) 15 (D) 20 (E) 25

5. A "domino" is made up of two small squares: .
 Which of the "checkerboards" illustrated below CANNOT be covered exactly and completely by a whole number of non-overlapping dominoes?

(A) 3×4 (B) 3×5 (C) 4×4 (D) 4×5 (E) 6×3 

6. Which number in the array below is both the largest in its column and the smallest in its row? (Columns go up and down, rows go right and left.)

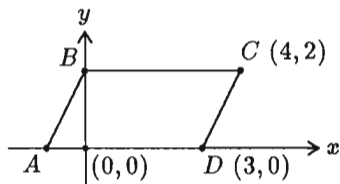
10	6	4	3	2
11	7	14	10	8
8	3	4	5	9
13	4	15	12	1
8	2	5	9	3

- (A) 1 (B) 6 (C) 7 (D) 12 (E) 15

7. The value of $\frac{(487,000)(12,027,300) + (9,621,001)(487,000)}{(19,367)(.05)}$ is closest to
(A) 10,000,000 (B) 100,000,000 (C) 1,000,000,000
(D) 10,000,000,000 (E) 100,000,000,000
8. What is the largest quotient that can be formed using two numbers chosen from the set $\{-24, -3, -2, 1, 2, 8\}$?
(A) -24 (B) -3 (C) 8 (D) 12 (E) 24
9. How many whole numbers from 1 through 46 are divisible by either 3 or 5 or both?
(A) 18 (B) 21 (C) 24 (D) 25 (E) 27

10. The area in square units of the region enclosed by parallelogram $ABCD$ is

(A) 6 (B) 8 (C) 12
(D) 15 (E) 18



11. There are several sets of three different numbers whose sum is 15 which can be chosen from $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. How many of these sets contain a 5?
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

12. If $\frac{2 + 3 + 4}{3} = \frac{1990 + 1991 + 1992}{N}$, then $N =$

(A) 3 (B) 6 (C) 1990 (D) 1991 (E) 1992

13. How many zeros are at the end of the product

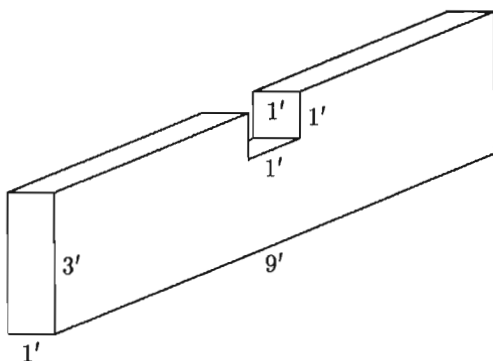
$$25 \times 25 \times 25 \times 25 \times 25 \times 25 \times 25 \times 8 \times 8 \times 8?$$

(A) 3 (B) 6 (C) 9 (D) 10 (E) 12

14. Several students are competing in a series of three races. A student earns 5 points for winning a race, 3 points for finishing second and 1 point for finishing third. There are no ties. What is the smallest number of points that a student must earn in the three races to be guaranteed of earning more points than any other student?

(A) 9 (B) 10 (C) 11 (D) 13 (E) 15

15. All six sides of a rectangular solid were rectangles. A one-foot cube was cut out of the rectangular solid as shown. The total number of square feet in the surface of the new solid is how many more or less than that of the original solid?



(A) 2 less (B) 1 less
(C) the same
(D) 1 more (E) 2 more

16. The 16 squares on a piece of paper are numbered as shown in the diagram. While lying on a table, the paper is folded in half four times in the following sequence:

- (1) fold the top half over the bottom half
- (2) fold the bottom half over the top half
- (3) fold the right half over the left half
- (4) fold the left half over the right half.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Which numbered square is on top after step 4?

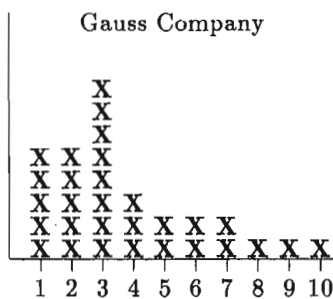
(A) 1 (B) 9 (C) 10 (D) 14 (E) 16

17. An auditorium with 20 rows of seats has 10 seats in the first row. Each successive row has one more seat than the previous row. If students taking an exam are permitted to sit in any row, but not next to another student in that row, then the maximum number of students that can be seated for an exam is

(A) 150 (B) 180 (C) 200 (D) 400 (E) 460

18. The vertical axis indicates the number of employees, but the scale was accidentally omitted from this graph. What percent of the employees at the Gauss Company have worked there for 5 years or more?

- (A) 9% (B) $23\frac{1}{3}\%$
 (C) 30% (D) $42\frac{6}{7}\%$
 (E) 50%



Number of years with company

19. The average (arithmetic mean) of 10 different positive whole numbers is 10. The largest possible value of any of these numbers is

- (A) 10 (B) 50 (C) 55 (D) 90 (E) 91

20. In the addition problem, each digit has been replaced by a letter. If different letters represent different digits then $C =$

- (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

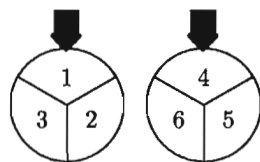
$$\begin{array}{r} A B C \\ B \\ + A \\ \hline 3 0 0 \end{array}$$

21. For every 3° rise in temperature, the volume of a certain gas expands by 4 cubic centimeters. If the volume of the gas is 24 cubic centimeters when the temperature is 32° , what was the volume of the gas in cubic centimeters when the temperature was 20° ?

- (A) 8 (B) 12 (C) 15 (D) 16 (E) 40

22. Each spinner is divided into 3 equal parts. The results obtained from spinning the two spinners are multiplied. What is the probability that this product is an even number?

- (A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{7}{9}$ (E) 1



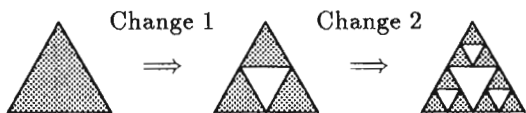
23. The Pythagoras High School band has 100 female and 80 male members. The Pythagoras High School orchestra has 80 female and 100 male members. There are 60 females who are members in both band and orchestra. Altogether, there are 230 students who are in either band or orchestra or both. The number of males in the band who are NOT in the orchestra is

(A) 10 (B) 20 (C) 30 (D) 50 (E) 70

24. A cube of edge 3 cm is cut into N smaller cubes, not all the same size. If the edge of each of the smaller cubes is a whole number of centimeters, then $N =$

(A) 4 (B) 8 (C) 12 (D) 16 (E) 20

25. An equilateral triangle is originally painted black. Each time the triangle is changed, the middle fourth of each black triangle turns white. After five changes, what fractional part of the original area of the black triangle remains black?



(A) $\frac{1}{1024}$ (B) $\frac{15}{64}$ (C) $\frac{243}{1024}$ (D) $\frac{1}{4}$ (E) $\frac{81}{256}$

SOLUTIONS

Your School Examination Manager has at least one copy of the 1991 AJHSME Solutions Pamphlet. It is meant to be lent to students (but not duplicated).

WRITE TO US!

Questions and comments about the problems and solutions for this AJHSME (but not requests for the Solutions Pamphlet) should be addressed to:

Mr Bruce Brombacher, AJHSME Chairman
Jones Middle School
Upper Arlington, OH 43221

Comments about administrative arrangements and orders for any publications listed below should be addressed to:

Prof Walter E Mientka, AMC Executive Director
Department of Mathematics and Statistics
University of Nebraska, Lincoln, NE 68588-0658

1992 AHSME

The American High School Mathematics Examination [AHSME] is a 30-question, 90-minute, multiple choice examination. Schools with high-scoring students on the AJHSME will receive a 1992 AHSME Invitation Brochure containing information about the AHSME and registration procedure. The best way to prepare for the AHSME is to study the exams from previous years. The procedure used to purchase these publications is indicated below.

PUBLICATIONS

MINIMUM ORDER: \$5 (before handling fee), US FUNDS ONLY. Canada and US orders must be prepaid. Orders are mailed 4th class, unless you specify 1st class, in which case add \$3 or 20% of total order, whichever is larger, with a maximum of \$15. Make checks payable to the American Mathematics Competitions.

FOREIGN ORDERS: do NOT prepay; an invoice will be sent.

COPYRIGHT: all publications are copyrighted; it is illegal to make copies without permission.

Examinations: Each price is for an examination and its solutions for one year. Specify the years you want and how many copies of each. All prices effective to July 1, 1992.

- AJHSME (Junior High Exam), 1985-1991, \$1 per copy per year.
- AHSME 1972-91, \$1 per copy per year.
- AIME 1983-91, \$2 per copy per year.
- AJHSME Summary of Results and Awards, 1985-90, \$4 per copy per year.
- AHSME Summary of Results and Awards, 1980-91, \$4 per copy per year.

Books (Exams and solutions):

- Problem Book I, AHSMEs 1950-60, \$8.50.
- Problem Book II, AHSMEs 1961-65, \$8.50.
- Problem Book III, AHSMEs 1966-72, \$10.
- Problem Book IV, AHSMEs 1973-82, \$11.
- USA Mathematical Olympiad Book 1972-86, \$13.
- International Mathematical Olympiad Book I, 1959-77, \$10.
- International Mathematical Olympiad Book II, 1978-85, \$11.