AMERICAN MATHEMATICS COMPETITIONS

AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

6th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 29, 1990

Sponsored by

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This Solutions Pamphlet gives at least one solution for each problem on this year's Examination and shows that all the problems can be solved using material normally associated with the seventh and eighth grade mathematics curriculum. The solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

It is hoped that teachers will find the time to share these solutions with their students.

Questions and comments about the problems and solutions (but **not** requests for the Solutions Pamphlet) should be addressed to:

Professor Thomas Butts, AJHSME Chairman
Science Education Department
The University of Texas at Dallas
PO Box 830688 FN32
Richardson, TX 75083-0688

Orders for prior year Examination questions and Solutions Pamphlets or Problem Books should be addressed to:

Prof Walter E Mientka, AMC Executive Director
Department of Mathematics and Statistics
University of Nebraska
Lincoln, NE 68588-0322

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1. C In the smallest such sum, the two smallest digits are in the hundred's places, the next two digits in the ten's places and the two largest digits are in the one's places. One example is $468 + 579 = 1047$.

   Query: In how many ways can this sum of 1047 be achieved?

2. A An increase in the tenth's place gives a larger value than an increase in any of the other decimal places. Since 1 is in the tenth's place of .12345, (A) is correct.

3. E Each shaded piece above or below the diagonal is matched by an identical unshaded piece meaning $\frac{1}{2}$ of the total area is shaded.

4. E From the way we multiply whole numbers, the unit's digit of the square of a whole number is determined by the square of the unit's digit of that whole number. The possible squares of unit's digits are: $0^2 = \boxed{0}$, $1^2 = \boxed{1}$, $2^2 = \boxed{4}$, $3^2 = \boxed{9}$, $4^2 = 1\boxed{6}$, $5^2 = 2\boxed{5}$, $6^2 = 3\boxed{6}$, $7^2 = 4\boxed{9}$, $8^2 = 6\boxed{4}$, $9^2 = 8\boxed{1}$.
   Note that 2,3,7, or 8 will never occur as the unit's digit of a square.

5. B The desired product is about $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} = 0.125$.

   OR

   The desired product is about $(.5)(.5)(.5) = 0.125$.

6. D All of the choices,except (C) and (D), are near 13,579. In (D), the result is the product (13579)(2468) while in (C) the result is much less than 13,579.

7. C For the product of three numbers to be positive, either all three of the numbers must be positive or one must be positive and two must be negative. Since there are only two positive numbers, only the latter case is possible. Thus the largest such product is $(-3)(-2)(5) = 30$.

8. D The sale price was $\frac{3}{4}(\$80) = \$60$. Thus the tax was \$6 and the total selling price was \$66.

9. D Five of the fifteen scores [77, 75, 84, 78, 80] are in the "C range", so the desired percent is $\frac{5}{15} = \frac{1}{3} = 33\frac{1}{3}\%$

10. A Since the date behind C is one less than that behind A, the date behind the desired letter must be one more than that behind B. This date is behind P.

11. E Since 11, 12, 13, 14, and 15 are on five of the faces, the number on the remaining face must be 10 or 16. In the first case, 10 must be on the face opposite the face with 15 which is impossible since $15 + 10 = 25$ would force 14 and 11 to be on opposite faces. Thus 16 is opposite 11, 12 is opposite 15, and 14 is opposite 13. The sum on each pair of opposite faces is 27, and the desired sum, is $3 \times 27 = 81$.

12. B One-fourth of the 24 numbers in the list begin with each of the four given digits. Those in positions 1 - 6 begin with 2; those in positions 7 - 12 begin with 4; those in positions 13 - 18 begin with 5. Thus the desired number is the fifth one beginning with 5: 5247, 5274, 5427, 5472, $\boxed{5724}$, 5742.

13. C The first ounce costs \$ .30. The additional 3.5 ounces would cost 4(\$ .22) = \$ .88. Thus the postage was \$ .30 + \$ .88 = \$1.18.

14. B Since one quarter of the balls are blue and there are 6 blue balls, there must be 24 balls in the bag. Thus there are $24 - 6 = 18$ green balls.

OR

Since one quarter of the balls are blue, three quarters of them must be green. Thus there are three times as many green balls as blue balls, so there are $3 \times 6 = 18$ green balls.

15. E The total area of the four squares is 100 cm^2, so the area of each square is 25 cm^2. Thus the side of each square is 5 cm and the perimeter of the figure is 10(5 cm) = 50 cm.

16. D By grouping as shown below, there are $\frac{199+1}{2} = 100$ groups of 10 for a sum of 1000:

$[1990 - 1980] + [1970 - 1960] + \ldots + [30 - 20] + 10$

17. A The number of cubic feet is $3 \times 60 \times \frac{1}{4} = 45$. Since there are 27 cubic feet in 1 cubic yard, there are $\frac{45}{27} = 1\frac{2}{3}$ cubic yards of concrete required. Thus 2 cubic yards must be ordered.

OR

Since 3 feet = 1 yard, 60 feet = 20 yards. Also 3 inches = $\frac{3}{36} = \frac{1}{12}$ yard. Thus $1 \times 20 \times \frac{1}{12} = 1\frac{2}{3}$ cubic yards of concrete are needed, so 2 cubic yards must be ordered.

18. C The original prism had 12 edges. Each "cut-off" corner yields 3 additional edges, so the new figure has a total of $12 + 8 \times 3 = 36$ edges.

19. B In order for the fewest number of seats to be occupied, there must be someone in every third seat, beginning with #2 and ending with #119. There are a total of $\frac{120}{3} = 40$ occupied seats.

OR

Consider some simpler cases and make a table:

Number of seats in the row:	3	6	9	12
Number of occupied seats in the row:	1	2	3	4

In each case, the middle seat in every group of three seats must be occupied, so the desired number of occupied seats in a row of 120 seats is $\frac{120}{3} = 40$.

20. A The difference between the incorrect sum and the actual sum is \$980,000 – \$98,000 = \$882,000. Since this difference is equally shared by all 1000 families, the difference between the means is $\frac{\$882000}{1000} = \$882$.

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21. B Working backward from 1024, divide each number [e.g. 1024] by the preceding number [e.g. 64] to get the previous number [e.g. 16] in the list. Thus $64 \div 16 = 4$, $4 \div 4 = 1$, and so on:

$$\underline{\tfrac{1}{4}}, \underline{4}, \underline{1}, \underline{4}, \underline{4}, \underline{16}, \underline{64}, \underline{1024}$$

22. B Since Chris takes the last piece of candy, each person receives the same number of the other 99 pieces of candy. Thus the number of students at the table must be a factor of 99. Only (B) fulfills this condition.

23. B The rate will the largest when the graph is the "steepest". During the second hour the distance traveled is about 500 miles, so the average speed during that hour is about 500 mph. For the other hours, the speeds are less than 350 mph.

24. C The second balance shows each Δ balances ◊ •. Replace each Δ on the first balance with ◊ •. Then after removing three •'s from each side, the balance has ◊◊◊◊ on the left and •••••• on the right. Thus ◊◊ will be balanced by •••.

25. C There are 8. Be systematic. You can begin with the five cases that have one corner square. Then consider the other three cases that do not have a corner square.