

AMERICAN MATHEMATICS COMPETITIONS

6th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL
MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 29, 1990

Sponsored by

Mathematical Association of America
Society of Actuaries Mu Alpha Theta
National Council of Teachers of Mathematics
Casualty Actuarial Society American Statistical Association
American Mathematical Association of Two-Year Colleges
American Mathematical Society

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. For each question, indicate your answer by marking the appropriate space on the answer form provided by your proctor.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. Use a #2 pencil since your answer form will be read by a marked-sense machine. Scratch paper, graph paper, rulers and erasers are permitted. Calculators may be used on the 1990 AJHSME. The nature of the questions has not changed from past tests. No problems on the test will require the use of a calculator, but you may have one available to use if you wish.
6. Unless specified otherwise, figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working the problems. You will have **40 MINUTES** working time for the test.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. What is the smallest sum of two 3-digit numbers that can be obtained by placing each of the six digits 4,5,6,7,8,9 in one of the six boxes in this addition problem?

A) 947 B) 1037 C) 1047 D) 1056 E) 1245

$$\begin{array}{r} \square \square \square \\ + \square \square \square \\ \hline \end{array}$$

2. Which digit of .12345, when changed to 9, gives the largest number?

A) 1 B) 2 C) 3 D) 4 E) 5

3. What fraction of the square is shaded?

A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{5}{12}$ D) $\frac{3}{7}$ E) $\frac{1}{2}$



4. Which of the following could **not** be the unit's digit [one's digit] of the square of a whole number?

A) 1 B) 4 C) 5 D) 6 E) 8

5. Which of the following is closest to the product $(.48017)(.48017)(.48017)$?

A) 0.011 B) 0.110 C) 1.10 D) 11.0 E) 110

6. Which of these five numbers is the largest?

A) $13579 + \frac{1}{2468}$ B) $13579 - \frac{1}{2468}$ C) $13579 \times \frac{1}{2468}$
 D) $13579 \div \frac{1}{2468}$ E) 13579.2468

7. When three different numbers from the set $\{-3, -2, -1, 4, 5\}$ are multiplied, the largest possible product is

A) 10 B) 20 C) 30 D) 40 E) 60

8. A dress originally priced at \$80 was put on sale at 25% off. If 10% tax was added to the sale price, then the total selling price of the dress was

A) \$45 B) \$52 C) \$54 D) \$66 E) \$68

9. The grading scale shown is used at Jones Junior High. The fifteen scores in Mr. Freeman's class were: 89, 72, 54, 97, 77, 92, 85, 74, 75, 63, 84, 78, 71, 80, 90.

A: 93 - 100
B: 85 - 92
C: 75 - 84
D: 70 - 74
F: 0 - 69

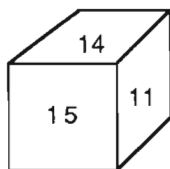
In Mr. Freeman's class, what percent of the students received a grade of C?

- A) 20% B) 25% C) 30% D) $33\frac{1}{3}\%$ E) 40%
10. On this monthly calendar, the date behind one of the letters is added to the date behind C. If this sum equals the sum of the dates behind A and B, then the letter is

	Tues.	Wed.	Thurs.	Fri.	Sat.
			C	A	
			Q		
	S	B	P	T	R

- A) P B) Q C) R D) S E) T

11. The numbers on the faces of this cube are consecutive whole numbers. The sums of the two numbers on each of the three pairs of opposite faces are equal. The sum of the six numbers on this cube is



- A) 75 B) 76 C) 78 D) 80 E) 81

12. There are twenty-four 4-digit whole numbers that use each of the four digits 2, 4, 5, and 7 exactly once. Listed in numerical order from smallest to largest, the number in the 17th position in the list is

- A) 4527 B) 5724 C) 5742 D) 7245 E) 7524

13. One proposal for new postage rates for a letter was 30¢ for the first ounce and 22¢ for each additional ounce (or fraction of an ounce). The postage for a letter weighing 4.5 ounces was

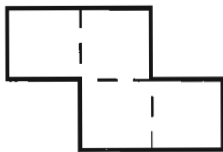
- A) 96¢ B) \$1.07 C) \$1.18 D) \$1.20 E) \$1.40

14. A bag contains only blue balls and green balls. There are 6 blue balls. If the probability of drawing a blue ball at random from this bag is $\frac{1}{4}$, then the number of green balls in the bag is

- A) 12 B) 18 C) 24 D) 30 E) 36

15. The area of this figure is 100 cm^2 .
Its perimeter is

A) 20 cm B) 25 cm C) 30 cm
D) 40 cm E) 50 cm



[figure consists
of four identical
squares]

16. $1990 - 1980 + 1970 - 1960 + \dots - 20 + 10 =$

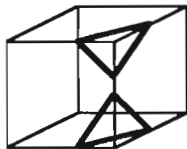
A) -990 B) -10 C) 990 D) 1000 E) 1990

17. A straight concrete sidewalk is to be 3 feet wide, 60 feet long and 3 inches thick. How many cubic yards of concrete must a contractor order for the sidewalk if concrete must be ordered in a whole number of cubic yards?

A) 2 B) 5 C) 12 D) 20 E) more than 20

18. Each corner of a rectangular prism is cut off. Two (of the eight) cuts are shown. How many edges does the new figure have?

A) 24 B) 30 C) 36 D) 42 E) 48



19. There are 120 seats in a row. What is the fewest number of seats that must be occupied so the next person to be seated must sit next to someone?

A) 30 B) 40 C) 41 D) 60 E) 119

20. The annual incomes of 1,000 families range from \$8200 to \$98,000. In error, the largest income was entered on the computer as \$980,000. The difference between the mean of the incorrect data and the mean of the actual data is

A) \$882 B) \$980 C) \$1078 D) \$482,000 E) \$882,000

21. A list of 8 numbers is formed by beginning with two given numbers. Each new number in the list is the product of the two previous numbers. Find the first number if the last three are shown:

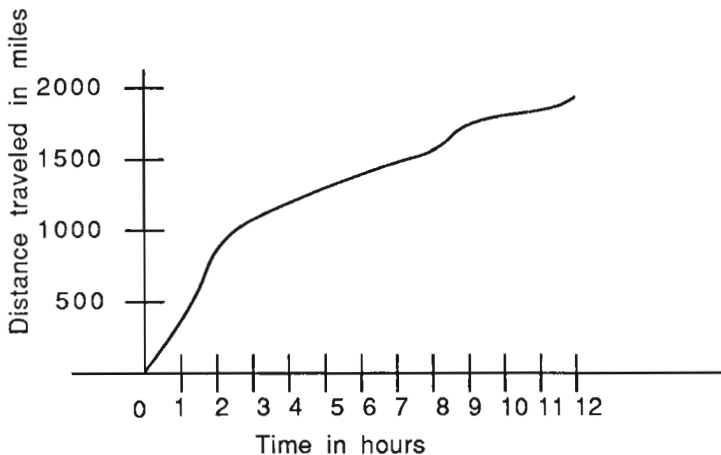
 , , , , , 16, 64, 1024

A) $\frac{1}{64}$ B) $\frac{1}{4}$ C) 1 D) 2 E) 4

22. Several students are seated at a large circular table. They pass around a bag containing 100 pieces of candy. Each person receives the bag, takes one piece of candy and then passes the bag to the next person. If Chris takes the first and the last piece of candy, then the number of students at the table could be

A) 10 B) 11 C) 19 D) 20 E) 25

23. The graph relates the distance traveled [in miles] to the time elapsed [in hours] on a trip taken by an experimental airplane. During which hour was the average speed of this airplane the largest?



- A) first (0-1) B) second (1-2) C) third (2-3) D) ninth (8-9) E) last (11-12)
24. Three Δ 's and a \diamond will balance nine \bullet 's. One Δ will balance a \diamond and a \bullet .



How many \bullet 's will balance the two \diamond 's in this balance?

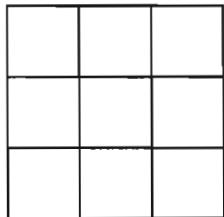
- A) 1 B) 2 C) 3 D) 4 E) 5



25. How many different patterns can be made by shading exactly two of the nine squares? Patterns that can be matched by flips and/or turns are not considered different. For example, the patterns shown below are **not** considered different.



- A) 3 B) 6 C) 8 D) 12 E) 18



SOLUTIONS

Your School Examination Manager has at least one copy of the 1990 AJHSME Solutions Pamphlet. It is meant to be lent to students (but not duplicated).

WRITE TO US!

Questions and comments about the problems and solutions for this AJHSME (but not requests for the Solutions Pamphlet) should be addressed to:

Prof Thomas Butts, AJHSME Chairman
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The University of Texas at Dallas
P O Box 830688 FN32, Richardson, TX 75083-0688

Comments about administrative arrangements and orders for any publications listed below should be addressed to:

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1991 AHSME

The American High School Mathematics Examination [AHSME] is a 30-question, 90-minute, multiple choice examination. Schools with high-scoring students on the AJHSME will receive a 1991 AHSME Invitation Brochure containing information about the AHSME and registration procedure. The best way to prepare for the AHSME is to study the exams from previous years. The procedure used to purchase these publications is indicated below.

PUBLICATIONS

MINIMUM ORDER: \$5 (before handling fee), US FUNDS ONLY. Canada and US orders must be prepaid. Orders are mailed 4th class, unless you specify 1st class, in which case add \$3 or 20% of total order, whichever is larger, with a maximum of \$15. Make checks payable to the American Mathematics Competitions.

FOREIGN ORDERS: do NOT prepay; an invoice will be sent.

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Examinations: Each price is for an examination and its solutions for one year. Specify the years you want and how many copies of each. All prices effective to July 1, 1991.

- **AJHSME** (Junior High Exam), 1985-1990, \$1 per copy per year.
- **Bulk AJHSME Sets**, \$5 per set. A set consists of ten copies of the 1985, 1986, 1987, 1988, 1989, or 1990 AJHSME.
- **AHSME 1972-90**, \$1 per copy per year.
- **AIME 1983-90**, \$2 per copy per year.
- **AJHSME Summary of Results and Awards, 1985-89**, \$4 per copy per year.
- **AHSME Summary of Results and Awards, 1980-90**, \$4 per copy per year.

Books (Exams and solutions):

- Problem Book I, AHSMEs 1950-60, \$8.50.
- Problem Book II, AHSMEs 1961-65, \$8.50.
- Problem Book III, AHSMEs 1966-72, \$10.
- Problem Book IV, AHSMEs 1973-82, \$11.