AMERICAN MATHEMATICS COMPETITIONS

AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

5th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, NOVEMBER 30, 1989

Sponsored by

Mathematical Association of America
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This Solutions Pamphlet gives at least one solution for each problem on this year's Examination and shows that all the problems can be solved using material normally associated with the seventh and eighth grade mathematics curriculum. The solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

It is hoped that teachers will find the time to share these solutions with their students.

Questions and comments about the problems and solutions (but **not** requests for the Solutions Pamphlet) should be addressed to:

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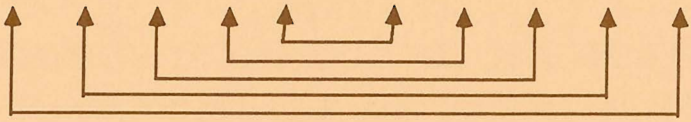
SOLUTIONS 1989 AJHSME

1. E The sum is 5(50) = 250 as can be seen by grouping the sum as shown.

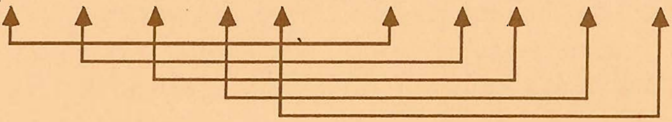
(1 + 11 + 21 + 31 + 41) + (9 + 19 + 29 + 39 + 49)



OR

The sum is 10 + 30 + 50 + 70 + 90 = 250 as can be seen by grouping the sum as shown.

(1 + 11 + 21 + 31 + 41) + (9 + 19 + 29 + 39 + 49)



2. D The sum is .2 + .04 + .006 = .246.

3. A We can compare the five numbers in the set by annexing zeroes so each one has four decimal places. The numbers, then, are .9900, .9099, .9000, .9090, .9009 so that .99 = .9900 is the largest.

4. E $\frac{401}{.205} \approx \frac{400}{.2} = \frac{4000}{2} = 2000$ where ≈ means "is approximately equal to"

OR

$$\frac{401}{.205} = \frac{1}{.205} \times 401 \approx 5 \times 400 = 2000$$

5. D Using the standard order of operations,

$-15 + 9 \times (6 \div 3) = -15 + 9 \times 2$ [operate inside parentheses]

$= -15 + 18$ [multiplication precedes addition]

$= 3.$

6. C Since the distance between two consecutive marks is $\frac{1}{5}$ of the total distance,

$y = 3 \times (\frac{1}{5} \times 20) = 3 \times 4 = 12.$

OR

Use a proportion: $\frac{y}{20} = \frac{3}{5}$, so $y = 12$.

7. D The value of 20 quarters and 10 dimes is \$5.00 + \$1.00 = \$6.00. Since the value of 10 quarters is \$2.50, the remaining \$3.50 must consist of 35 dimes.

OR

We can think of the 20 – 10 = 10 quarters as being "traded in" for dimes. The value of 10 quarters, \$2.50, is 25 dimes. Combining them with the original 10 dimes gives a total of 35 dimes.

8. E Using the distributive law, the product equals

$(2 \times 3 \times 4) \times (\frac{1}{2}) + (2 \times 3 \times 4) \times (\frac{1}{3}) + (2 \times 3 \times 4) \times (\frac{1}{4}) =$

$\{3 \times 4\} + \{2 \times 4\} + \{2 \times 3\} = 12 + 8 + 6 = 26.$

Although it is more tedious, one can perform the operations within each set of parentheses to obtain:

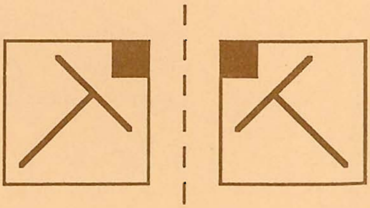
$24\left(\frac{6}{12} + \frac{4}{12} + \frac{3}{12}\right) = 24\left(\frac{13}{12}\right) = 26.$

9. C We see that 2 out of every 5 students or $\frac{2}{5}$ or 40% are boys.

Note: The number of students in the class is not needed to solve the problem.

10. D The numerals on a clock face divide it into twelve equal sections of 30°. The smaller of the two regions determined by the hands at 7:00 p.m. covers five of these sections, so the desired angle is 150°.

11. B Two figures are symmetric with respect to a line if the figures coincide when the paper is folded along that line.



12. E $$\frac{1-\frac{1}{3}}{1-\frac{1}{2}} = \frac{\frac{2}{3}}{\frac{1}{2}} = \frac{2}{3} \times \frac{2}{1} = \frac{4}{3}.$$

OR

Multiplying both numerator and denominator by 6 yields

$$\frac{6(1-\frac{1}{3})}{6(1-\frac{1}{2})} = \frac{6-2}{6-3} = \frac{4}{3}.$$

13. A To get .9 in the numerator, we must divide it by 10. To maintain equality, we must also divide the denominator by 10. Thus

$$\frac{\frac{9}{10}}{\frac{7 \times 53}{10}} = \frac{\frac{9}{10}}{\frac{7}{10} \times 53} = \frac{.9}{.7 \times 53}.$$

In all other cases, the resulting fraction is $\frac{1}{10}$ or $\frac{1}{100}$ of the original fraction.

14. C The smallest difference occurs when the minuend is as small as possible and the subtrahend is as large as possible. Thus $245 - 96 = 149$ is the smallest difference.

15. D The shaded area is the difference of the area of the parallelogram and the area of the unshaded triangle. The area of the parallelogram is $10 \times 8 = 80$. The base of the triangle is $10 - 6 = 4$ and its height is 8, so its area is $\frac{1}{2} \times 4 \times 8 = 16$. Thus the shaded area is $80 - 16 = 64$.

OR

The shaded region is a trapezoid with bases 10 and 6, and height 8. Its area is $\frac{1}{2}(8)(10+6) = 64$.

16. A It is not possible to write 47 as the sum of two primes. When 47 is written as the sum of two whole numbers, one must be even and the other odd. Since 2 is the only even prime, there is only one case to consider. In that case, the other summand must be 45 which is not a prime since it has a factor of 5.

17. B Since N is between 9 and 17, the average $\frac{6 + 10 + N}{3}$ must lie between $\frac{6 + 10 + 9}{3} = 8\frac{1}{3}$ and $\frac{6 + 10 + 17}{3} = 11$. Only 10 lies in this range.

OR

The average of $6 = 10 - 4$, 10, and $14 = 10 + 4$ is clearly 10. Since $9 < 14 < 17$, a possible average is 10. Since the question has only one answer, it must be 10.

18. B Depressing the key one time yields the reciprocal of 32 or $\frac{1}{32}$. Thus depressing the key a second time yields the reciprocal of $\frac{1}{32}$ or $\frac{1}{\frac{1}{32}} = 32$.

19. B The graph shows expenditures of a bit more than \$2 million by the beginning of June and a bit more than \$4.5 million by the end of August. Thus about $4.5 - 2.0 = \$2.5$ million was spent during the summer months.

20. D Each number will share a corner with every number on the cube except the one on the opposite face. Thus the combinations involving 1-3, 2-5, and 4-6 cannot occur. Consequently the largest sum is $6 + 5 + 3 = 14$.

21. D Since $25\% = .25 = \frac{1}{4}$, Jill bought $\frac{1}{4} \times 128 = 32$ apples. June bought $\frac{1}{4}$ of the remaining $128 - 32 = 96$ apples or 24 apples. This leaves 72 apples -- one for the teacher and 71 for Jack.

OR

Keep track of the number of apples that Jack has at the end of each transaction:

Jill: $\frac{3}{4} \times 128 = 96$

June: $\frac{3}{4} \times 96 = 72$

Teacher: $72 - 1 = 71$.

22. C The 6 letters will go through a complete cycle every 6 lines and the 4 numbers every 4 lines. The letters and numbers together will go through a complete cycle every LCM $(6,4) = 12$ times. Thus AJHSME 1989 occurs for the first time on the 12th line.

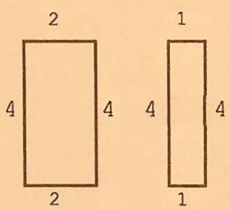
23. C There are 6 faces exposed on each of the four sides for a total of $6 \times 4 = 24$ square meters. From above, there are a total of 9 square meters exposed since the cubes in the second and third layers essentially cover the unexposed portion of the bottom layer. So there are a total of $24 + 9 = 33$ square meters to paint.

OR

The bottom layer has 12 complete faces, four "$\frac{1}{2}$ faces", and four "$\frac{3}{4}$ faces". The second layer has 8 complete faces and four "$\frac{3}{4}$ faces". The top layer has 5 complete faces. The total, then, is

$$12 + 4\left(\frac{1}{2}\right) + 4\left(\frac{3}{4}\right) + 8 + 4\left(\frac{3}{4}\right) + 5 = 33 \text{ square meters.}$$

24. E



If we let the side of the original square be 4 units, then the large rectangle and one of the small rectangles formed have the dimensions shown. Therefore the ratio of the perimeters is

$$\frac{1+4+4+1}{2+4+4+2} = \frac{10}{12} = \frac{5}{6}.$$

25. C The set of all the possible outcomes of spinning each wheel is
{ (5, 6) , (5, 7), (5, 9), (8,6), (8, 7), (8,9), (4,6), (4,7), (4,9), (3,6), (3,7), (3,9) }
There are six even sums among the 12 outcomes for a probability of $\frac{6}{12} = \frac{1}{2}$.

OR

Consider the outcome on the right wheel first. Whatever number occurs, there are two out of four chances for the number on the left wheel to give an even sum. Thus the desired probability is $\frac{2}{4} = \frac{1}{2}$.