AMERICAN MATHEMATICS COMPETITIONS

AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

4th ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
(AJHSME)

THURSDAY, DECEMBER 1, 1988

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Mathematical Association of America
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This Solutions Pamphlet contains at least one solution to each problem on this year's Examination and shows that all the problems can be solved using material normally associated with the seventh and eighth grade mathematics curriculum. The solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

We hope teachers will share these solutions with their students.

Questions and comments about the problems and solutions (but **not** requests for the Solutions Pamphlet) should be addressed to:

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Order prior year Examination questions and Solutions Pamphlets or Problem Books from:

1.   D   The scale is divided into fourths and the needle is just past the $\frac{1}{4}$ mark, so the reading must be between 10.25 and 10.5.

2.   C   The product can be rewritten as $8 \times \frac{1}{4} \times 2 \times \frac{1}{8} =$

$$8 \times \frac{1}{8} \times \frac{1}{4} \times 2 = \frac{1}{2}$$

3.   D   Since $\frac{2}{20} = \frac{3}{30} = \frac{1}{10}$, the desired sum is $.1 + .1 + .1 = .3$.

4.   B   In each row, including the first, there is one more dark square than light square. Since there are 8 rows, there must be 8 more dark squares than light squares.

5.   C   Since $\angle CBD$ is a right angle, side BC must cross the protractor at 70°. Thus the measure of $\angle ABC$ is $70° - 20° = 50°$.

OR

$\angle ABC = \angle ABD - \angle CBD = (\angle OBD - \angle OBA) - \angle CBD =$
$(160° - 20°) - 90° = 50°$.

6.   E $\dfrac{(.2)^3}{(.02)^2} = \dfrac{.008}{.0004} = \dfrac{80}{4} = 20$

OR

$\dfrac{.2}{.02} \times \dfrac{.2}{.02} \times .2 = 10 \times 10 \times .2 = 20.$

7.   B   The product is approximately $(2.5)(8)(10) = (20)(10) = 200$.

8.   B   Although one could solve this problem by counting decimal places in the product, it is more more insightful to realize that the answer is approximately $.1(2) = .2$, so (B) is correct.

9.   D   All but the triangle in the upper right are isosceles.

10.   A   60 days is 8 weeks and 4 days, so the desired day is four days after a Thursday , that is, a Monday.

<div align="center">OR</div>

                 60 days is 9 weeks less 3 days, so the desired day is three days before a Thursday , also a Monday.

11.   E $10^2 = 100$, $11^2 = 121$, $12^2 = 144$, $13^2 = 169$ so (E) is correct.

12.   C   The cost per person $= \dfrac{\text{total cost}}{\text{number of people}}$.

                 Thus $\dfrac{\$20 \text{ billion}}{250 \text{ million}} = \dfrac{2 \times 10^{10}}{2.5 \times 10^8} = .8 \times 10^2 = \$80.$

<div align="center">OR</div>

                 Since $\dfrac{1000000000}{250000000} = 4,$ the cost per person is ($4)(20) or $80.

13.   D   The circumference of the circular patio is $2\pi(12) \approx (2)(3.14)(12) \approx 75$ feet, thus it would take about 75 bushes to surround the patio.

14.   E   The factor pairs for 36 are 1x36, 2x18, 3x12, 4x9, and 6x6. The largest sum of such a pair is 37.
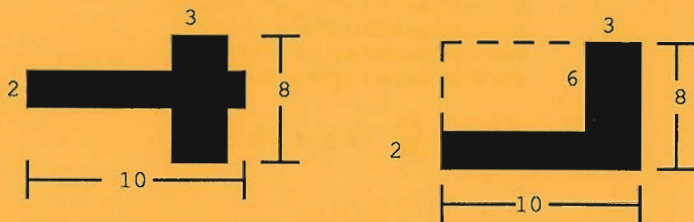
15.   C $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ and its reciprocal is $\frac{6}{5}$.

16.   E   The arrangement pictured shows 6 X's is possible. If there were 7 X's on a 3x3 board, then one row must contain 3 X's.

17.   B

The total shaded area is the sum of the areas of the "horizontal" rectangle and the "vertical" rectangle minus the area of the "overlapping" rectangle that is part of both of the other rectangles. Thus the desired area is $2(10) + 3(8) - 2(3) = 38$.

OR

"Slide" the rectangles as shown in the figure on the right so they form an L-shaped figure. We see that the shaded area is $(10)(2) + (6)(3) = 38$ or $(7)(2) + (8)(3) = 38$ or $(10)(8) - (7)(6) = 38$.

18.   C   The total weight of the ten children is $6(150) + 4(120) = 1380$, so the average weight is $\frac{1380}{10}$ or 138 pounds.

OR

The average weight of 4 boys and 4 girls is 135 pounds. The other two boys would raise this average by $\frac{30}{10}$ or 3 pounds.

19.   A   Adding 3 to each term in the original arithmetic sequence yields the sequence 4,8,12,16,20, ... in which the one-hundredth term is 400. Subtracting 3 from each term shows that 397 is the one-hundredth term of the original sequence.
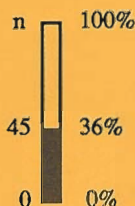
OR

We may obtain the 100th term in the sequence by adding 4 to the first term 1 a total of 99 times. Thus the 100th term is $1 + 99(4) = 397$.

20. C If n is the number of cups of coffee the coffeemaker will hold when it is full, then we can label the gauge as shown and write the two equivalent ratios:

$$\frac{45}{n} = \frac{36}{100} \quad \text{or} \quad n = 125 \text{ cups.}$$

n	100%
45	36%
0	0%

OR

If 36% is 45 cups, then 4% is $\frac{1}{9}(45)$ or 5 cups so

that 100% is $25(5)$ or 125 cups.

21. C There are three possible values for the new number, n. It is either less than 6, between 6 and 9, or greater than 9. If the five elements are listed in increasing order, these three possibilities in the table below are: one of the first two, the middle one, and one of the last two. Since the median is the middle number in a set of five elements, there are three values for the median: 6, n, or 9.

|					Median
n	3	6	9	10	6
3	n	6	9	10	6
3	6	n	9	10	n
3	6	9	n	10	9
3	6	9	10	n	9

Query: Can you find the values of n and which of the two alternatives occurs in the first and third cases ?

22. E An item whose original cost was $100, for example, will cost $25 more or $125. The sale price of a $125 item will be 80% of its current price or .8($125) = $100 - - the original cost. The same kind of comparison can be made for any original cost.

23. D   Since the computer disks are bought in groups of 4 and sold in groups of 3, it is easier to consider them in groups of 12 or dozens. Each dozen costs $15 and sells for $20 giving a profit of $5. Thus to get a profit of $100, she must sell 20 dozen or 240 computer disks.

24. A

1.   2.   3.   4.
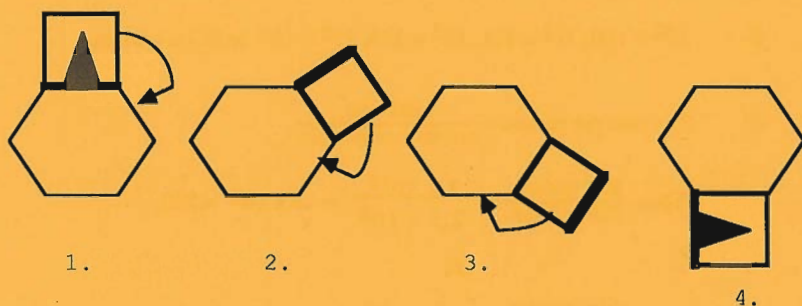
Keep track of the "bottom" side of the square in the first figure. In the fourth figure, it will appear on the left, so the solid triangle will be in the position shown in (A).

OR
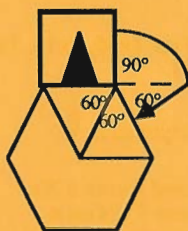
Each time the square "rolls" to the next edge of the hexagon, it turns through an angle of 150°. In going from the top to the bottom of the hexagon, the square makes three such turns for a total of 3(150) = 450°. This 450° represents one complete revolution and 1/4 of a second revolution.

25. A   In each hour from 1:00 through 9:59, there are six such times. From 3:00 to 3:59, for example, these times are 3:03, 3:13, 3:23, 3:33, 3:43, 3:53. From 10:00 to 12:59, there is one such time in each hour: 10:01, 11:11, and 12:21. Thus there are a total of 9(6) + 3 = 57 such times.