AJHSME SOLUTIONS PAMPHLET
FOR STUDENTS AND TEACHERS

3rd ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
1987

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This Solutions Pamphlet gives at least one solution for each problem on this year's Examination and shows that all the problems can be solved using material normally associated with the seventh and eighth grade mathematics curriculum. The solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

It is hoped that teachers will find the time to share these solutions with their students.

AMERICAN MATHEMATICS COMPETITIONS

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1. E The sum is .426.

2. B $\frac{2}{25} = \frac{8}{100} = .08.$

3. E Pairing the addends as shown, we see the desired product is (2)(5)(180) = 1800.

$$2(\,81 + 83 + 85 + 87 + 89 + 91 + 93 + 95 + 97 + 99\,)$$

4. C A right angle is 1/4 of a full circle, so there are $\frac{1}{4}(500) = 125$ clerts in a right angle.

5. A The area is $(.4\text{ m})(.22\text{ m}) = .088\text{ m}^2$.

6. B A negative product occurs when multiplying a positive number by a negative number. The minimum product will occur, then, when multiplying the smallest negative number by the largest positive number. In this case, that product is $(-7)(3) = -21$.

7. C The three faces shown of the larger cube must contain half the smaller shaded cubes, so there are $2(4 + 5 + 1) = 20$ shaded smaller cubes. By carefully looking at the large cube, we see that each smaller cube has at most one face (contained in a face of the large cube) shaded.

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8. B $7 + 3 + B > 9$, so we "carry 1" from the ten's column to the hundred's column. Similarly $1 + 8 + A > 9$ since $A > 0$, so we "carry 1" from the hundred's place to the thousand's place. $1 + 9 = 10$, so the sum is a 5-digit number of the form 10CD9.

OR

Since A is a digit from 1 to 9, the sum 9876 + A32 must be between 10,000 and 100,000. Thus the sum of the three whole numbers must have 5 digits.

9. C The least common denominator(LCD) is the least common multiple of the denominators 2,3,4,5,6,7 or $2 \cdot 2 \cdot 3 \cdot 5 \cdot 7 = 420$.

10. B The desired sum may be written as $299(4 + 3 + 2 + 1) - 1 = 299(10) - 1 = 2990 - 1 = 2989$.

11. B The desired sum can be rewritten as $2 + 3 + 5 + \frac{1}{7} + \frac{1}{2} + \frac{1}{19}$ which equals $10 + \frac{1}{2} + \left(\text{a number less than } \frac{1}{2}\right)$ so B is correct.

12. C The large rectangular region can be subdivided into 24 congruent rectangular regions of which 2 are shaded.

OR

$\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$ of the rectangular region is shaded.

13. E If the numerator of a fraction is less than half its denominator, then the value of the fraction is less than $\frac{1}{2}$. Consequently all the fractions other than (E) are less than $\frac{1}{2}$ while $\frac{151}{301} > \frac{1}{2}$.

14. B There are $60 \cdot 60 = 3600$ seconds in an hour. Thus the computer does $3600 \cdot 10{,}000 = 36{,}000{,}000$ or 36 million additions in an hour.

15. D The regular price for three tires is \$240 - \$3 = \$237. Thus the regular price of one tire is $\frac{\$237}{3} = \$79$.

16. E In order to have a seasonal shooting average of 50% when having attempted 30 + 10 = 40 shots, Joyce must have made 20 of them. Thus she made 8 of her 10 shots in the next game.

17. D Since Bret is in seat #3 and statement (1) is false, Carl must be in seat #1. Then, since statement (2) is false, Abby must be in seat #4 leaving Dana in seat #2.

18. C The 12 people not dancing are $\frac{2}{3}$ of the people remaining, so 18 people remained. Thus there were 2(18) = 36 people in the room originally.

19. A The next four numbers displayed are 4, 16, 256, and $256^2 \approx 60{,}000$. Thus 500 is exceeded on the fourth depression of the $\boxed{x^2}$ key.

20. A To show the statement is false, we must find a value of n so that n is not prime and n - 2 is prime. Such a value is n = 9.

21. C Statements i and ii are false; iii and iv are true.

i. $3^* + 6^* = \frac{1}{3} + \frac{1}{6} \neq \frac{1}{9}$ iii. $2^* \cdot 6^* = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12} = 12^*$

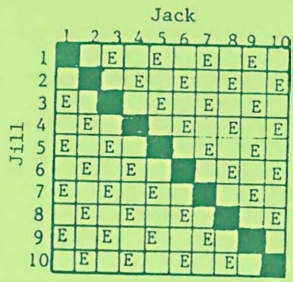
ii. $6^* - 4^* = \frac{1}{6} - \frac{1}{4} \neq \frac{1}{2}$ iv. $10^* \div 2^* = \frac{1}{10} \cdot \frac{2}{1} = \frac{1}{5} = 5^*$

22. D By the Pythagorean Theorem, AC = 5. Since AC = BD, the radius of the circle is 5. The area of the shaded region, then, is $\frac{1}{4}\cdot\pi\cdot 5^2 - 3\cdot 4$ (a quarter circle with a rectangle deleted) $= \frac{25\pi}{4} - 12$. This quantity is clearly greater than $\frac{76}{4} - 12 = 7$ and less than $\frac{25(3\frac{1}{5})}{4} - 12 = 8$ since $3 < \pi < 3\frac{1}{5}$.

23. D The total Black population is the sum of the Black populations in each of the four regions or $5 + 5 + 15 + 2 = 27$ million. Thus $\frac{15}{27} = \frac{5}{9} \approx 55.56\%$ lived in the South. (The population figures were taken from the World Almanac and adjusted slightly for convenience.)

24. D If John answered 13 or more questions correctly, then his score would have been at least $13(5) - 7(2) = 51$ (13 correct, 7 incorrect). Checking the other cases, we find that John could have answered 12 correctly, 6 incorrectly, and left 2 unanswered for a score of $12(5) - 6(2) = 48$. Note that 10 correct, 1 incorrect, 9 unanswered also give a score of 48.

25. A Since Jack and Jill cannot remove the same number, there are $10 \cdot 9 = 90$ ways they can remove the two balls from the jar as shown by the unshaded squares on the grid. Those squares representing an even sum are labeled "E". There are 40 such squares - 4 in each column (or row) since the two numbers must both be odd or both be even. The probability is $\frac{40}{90} = \frac{4}{9}$.



OR

There are 10 ways to select the first number but only 4 ways to select the second since it must have the same parity (both odd or both even) as the first. Thus the probability is $\frac{10 \cdot 4}{10 \cdot 9} = \frac{4}{9}$.