AJHSME SOLUTIONS PAMPHLET

FOR STUDENTS AND TEACHERS

2nd ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
1986

Sponsors:
MATHEMATICAL ASSOCIATION OF AMERICA
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This Solutions Pamphlet gives at least one solution for each problem on this year's Examination and shows that all the problems can be solved using material normally associated with the seventh and eighth grade mathematics curriculum. The solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

It is hoped that teachers will find the time to share these solutions with their students.

AMERICAN MATHEMATICS COMPETITIONS

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1. (A) The average rainfall per hour equals the total rainfall divided by the total number of hours. Since July has 31 days, the average is $\frac{366}{31\times 24}$ inches per hour.

2. (A) A large positive number has a small reciprocal and vice-versa. The smallest positive number $\frac{1}{3}$ has the largest reciprocal, $\frac{1}{\frac{1}{3}} = 3$.

3. (C) The three smallest numbers in the set are -3, -1, 7. Their sum is -3 + -1 + 7 = 3.

4. (C) The desired product is about 2(40) - .2(40) = 80 - 8 = 72, so (C) is correct.

5. (D) Since 1000 minutes = $\frac{1000}{60}$ hours = $16\frac{2}{3}$ hours = 16 hours, 40 minutes, the contest ended 16 hours 40 minutes past noon or at 4:40 a.m.

6. (E) $\frac{2}{1-\frac{2}{3}} = \frac{2}{\frac{1}{3}} = 2\times 3 = 6.$

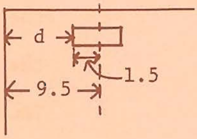
7. (B) Since $\sqrt{8} < \sqrt{9} = 3$, and $\sqrt{80} < \sqrt{81} = 9$, the desired whole numbers are 3,4,5,6,7,8; there are six of them.

8. (E) If B×2 ends in 6, then B is 3 or 8. Since the product exceeds 6000, B must be 8.

9. (E) The possible routes are MADCN, MACN, MBADCN, MBACN, MBCN, and MBN.

Query: Can you think of a systematic method to count the routes for more complicated figures?

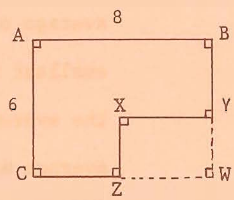
10. (B) Using the diagram, we see that the distance d = 9.5 feet - 1.5 feet = 8 feet.



11. (A) $(3*5)*8 = (\frac{3+5}{2})*8 = 4*8 = \frac{4+8}{2} = 6.$

12. (D) A student received the same grade on both tests if s/he is counted on the main diagonal (from the top left to the bottom right) of the table. Thus the number of students receiving the same grade on both tests is $2 + 4 + 5 + 1 + 0 = 12$. Consequently $\frac{12}{30} = \frac{4}{10} = 40\%$ of the students received the same grade on both tests.

13. (C) Since XY = ZW and XZ = YW, the perimeter of polygon ABYXZC is equal to the perimeter of rectangle ABWC, or $2(8 + 6) = 28$. Note that the solution to the problem does not depend on the position of the point X inside the rectangle.



14. (C) The maximum value for the quotient $\frac{b}{a}$ is formed by choosing the largest possible value for b and the smallest possible value for a, or $\frac{1200}{200} = 6$.

15. (B) The sale price of the coat is 50% of \$180 or \$90. The additional discount is 20% of \$90 or \$18, so the Saturday price is \$90 - \$18 = \$72.

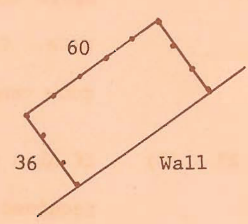
OR

The Saturday price is 80% of the sale price and the sale price is 50% of the original price, so the Saturday price is .8(.5(\$180)) = .8(\$90) = \$72.

16. (A) If the fall sales of 4 million hamburgers are 25% of the yearly sales, then the yearly sales are 16 million hamburgers. Thus the winter sales are $16 - (4.5 + 5 + 4) = 2.5$ million.

17. (E) The number o^2 is always odd. Now (no) is odd if n is odd and even if n is even. Thus the sum (o^2 + no) is even if n is odd, since the sum of two odd numbers is even; the sum is odd if n is even, since the sum of an odd number and an even number is odd.

18. (B) The fewest number of posts is used if the wall serves as the longer side of the rectangular grazing area. Thus there are 6 posts on the 60 meter side (including the corners) and 3 more posts on each 36 meter side for a total of 12 posts.



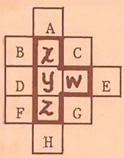
19. (D) The trip was 57,060 - 56,200 = 860 miles long and 12 + 20 = 32 gallons of gasoline were used during the trip. Thus the average number of miles per gallon was $\frac{860}{32} = 26.9$. Note that the 6 gallons needed to fill the tank at the start of the trip had no effect on the answer. No matter how many gallons were needed to fill the tank at the start, the average miles per gallon for the trip would be 26.9.

20. (D) Estimate each of the quantities to obtain the approximation:

$$\frac{(300)^5}{30(400)^4} = \frac{300}{30}\left(\frac{300}{400}\right)^4 = 10\left(\frac{3}{4}\right)^4 = 10\left(\frac{81}{256}\right)$$

$$\simeq 10\left(\frac{1}{3}\right) \approx 3.$$

21. (E) Label the four squares in the T-shaped figure X,Y,Z,W as shown. First think of W as the base of the cube. Then X,Y,Z will be three of the "sides" and the fourth side could be A, E, or H. Now think of Y as the base. Then X,W,Z are three sides and B, D, or F could be the fourth side. C and G are not possible because four sides of a cube cannot come together at a point.



22. (C) If Alan received an "A", then all the others would have received an "A". If Beth received an "A", then Carlos and Diana would also have received an "A". Thus only Carlos and Diana received an "A".

23. (B) The radius of the large circle is 2 since it is a diameter of a small circle. The area of the large circle is $\pi(2)^2 = 4\pi$ and the area of each small circle is π. The shaded area is (by symmetry) half the difference of the areas of the large circle and the two small circles, i.e. $\frac{1}{2}(4\pi - 2\pi) = \pi$. Thus the desired ratio is 1.

24. (B) Al must be assigned to one of the lunch groups. The probability that Bob is assigned to the same lunch group is approximately $\frac{1}{3}$ ($\frac{199}{599}$ exactly) and the probability that Carol is assigned to that same group is also approximately $\frac{1}{3}$ ($\frac{198}{598}$). Thus the probability that all three are assigned to the same group is approximately $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

25. (D) In a set of whole numbers which are equally spaced, the average of the numbers in the set is the average of the smallest number and the largest number. For example, the average of $\{ 1,3,5,7,9 \} = \frac{1+9}{2} = 5$, and the average of $\{ 2,5,8,11,14,17 \}$ is $\frac{2+17}{2} = 9.5$.

In this problem, then, the averages are:

A: $\frac{2+100}{2} = 51$, B: $\frac{3+99}{2} = 51$,

C: $\frac{4+100}{2} = 52$, D: $\frac{5+100}{2} = 52.5$,

E: $\frac{6+96}{2} = 51$.

One could "guesstimate" that the set with the "largest" numbers should have the largest average. The numbers 5 and 100 are (overall) larger than the corresponding numbers in the other sets.