AMERICAN MATHEMATICS COMPETITIONS

# 2nd ANNUAL
# AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
# (AJHSME)

## TUESDAY, DECEMBER 9, 1986

Sponsors:

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## INSTRUCTIONS AND INFORMATION

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This test contains twenty-five multiple choice questions. Each question is followed by five possible answers labeled A, B, C, D, and E. Only one answer is correct.
3. Several problems have only one reasonable answer—all others can be eliminated by estimating. Therefore, if a question seems to require a good deal of calculation, try estimating instead.
4. For each question, indicate your answer by marking the appropriate space on the answer card provided by your proctor.
5. There is no penalty for guessing. Your score on this test is the number of correct answers.
6. Use a #2 pencil since your answer card will be read by a marked-sense machine. Scratch paper, graph paper, rulers and erasers are permitted. *Calculators are not permitted.*
7. Unless specified otherwise, figures are not necessarily drawn to scale.
8. Before beginning the test, your proctor will ask you to record certain information on the answer card.
9. When your proctor gives the signal, begin to work the problems. You have **40 MINUTES** working time.

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The MAA Committee on the American Mathematics Competitions reserves the right to reexamine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. In July 1861, 366 inches of rain fell in Cherrapunji, India. What was the average rainfall in inches per hour during that month?

A) $\frac{366}{31\times24}$ B) $\frac{366\times31}{24}$ C) $\frac{366\times24}{31}$

D) $\frac{31\times24}{366}$ E) $366\times31\times24$

2. Which of the following numbers has the largest reciprocal?

A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) 1 D) 5 E) 1986

3. The smallest sum one could get by adding three different numbers from the set { 7, 25, -1, 12, -3 } is

A) -3 B) -1 C) 3 D) 5 E) 21

4. The product $(1.8)(40.3 + .07)$ is closest to

A) 7 B) 42 C) 74 D) 84 E) 737

5. A contest began at noon one day and ended 1000 minutes later. At what time did the contest end?

A) 10:00 p.m. B) midnight C) 2:30 a.m.
D) 4:40 a.m. E) 6:40 a.m.

6. $\frac{2}{1-\frac{2}{3}}$ =

A) -3 B) $-\frac{4}{3}$ C) $\frac{2}{3}$ D) 2 E) 6

7. How many whole numbers are between $\sqrt{8}$ and $\sqrt{80}$ ?

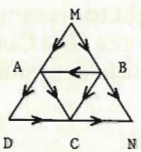
A) 5 B) 6 C) 7 D) 8 E) 9

8. In the product shown, B is a digit. The value of B is

$$\begin{array}{r} B2 \\ \times 7B \\ \hline 6396 \end{array}$$

A) 3 B) 5 C) 6 D) 7 E) 8

9. Using only the paths and the directions shown, how many different routes are there from M to N?




A) 2 B) 3 C) 4 D) 5 E) 6

10. A picture 3 feet across is hung in the center of a wall that is 19 feet wide. How many feet from the end of the wall is the nearest edge of the picture?

A) $1\frac{1}{2}$ B) 8 C) $9\frac{1}{2}$ D) 16 E) 22

11. If A*B means $\frac{A+B}{2}$, then (3*5)*8 is

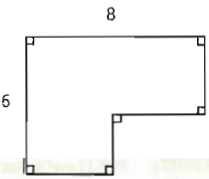
A) 6 B) 8 C) 12 D) 16 E) 30

12. The table to the right displays the grade distribution of the 30 students in a mathematics class on the last two tests. For example, exactly one student received a 'D' on Test 1 and a 'C' on Test 2 (see circled entry). What percent of the students received the same grade on both tests?

| TEST 1 \ TEST 2 | A | B | C | D | F |
|---|---|---|---|---|---|
| A | 2 | 2 | 1 | 0 | 0 |
| B | 1 | 4 | 3 | 0 | 0 |
| C | 1 | 3 | 5 | 2 | 0 |
| D | 0 | 0 | (1) | 1 | 1 |
| F | 0 | 0 | 2 | 1 | 0 |

A) 12% B) 25% C) $33\frac{1}{3}\%$ D) 40% E) 50%

13. The perimeter of the polygon shown is




A) 14 B) 20 C) 28 D) 48

E) cannot be determined from the information given

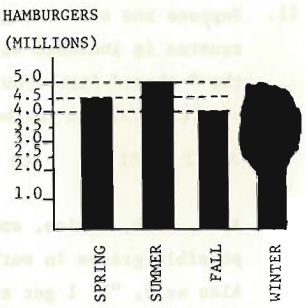
14. If $200 \leq a \leq 400$ and $600 \leq b \leq 1200$, then the largest value of the quotient $\frac{b}{a}$ is

A) $\frac{3}{2}$ B) 3 C) 6 D) 300 E) 600

15. Sale prices at the Ajax Outlet Store are 50% below original prices. On Saturdays an additional discount of 20% off the sale price is given. What is the Saturday price of a coat whose original price is \$180?

A) \$54 B) \$72 C) \$90 D) \$108 E) \$110

16. A bar graph shows the number of hamburgers sold by a fast food chain each season. However, the bar indicating the number sold during the winter is covered by a smudge. If exactly 25% of the chain's hamburgers are sold in the fall, how many million hamburgers are sold in the winter?




A) 2.5 B) 3 C) 3.5 D) 4 E) 4.5

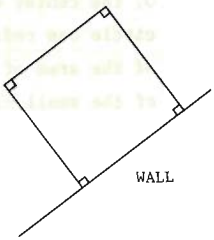
17. Let o be an odd whole number and let n be any whole number. Which of the following statements about the whole number $(o^2 + no)$ is always true?

A) it is always odd B) it is always even

C) it is even only if n is even D) it is odd only if n is odd

E) it is odd only if n is even

18. A rectangular grazing area is to be fenced off on three sides using part of a 100 meter rock wall as the fourth side. Fence posts are to be placed every 12 meters along the fence including the two posts where the fence meets the rock wall. What is the fewest number of posts required to fence an area 36 m by 60 m?




A) 11 B) 12 C) 13 D) 14 E) 16

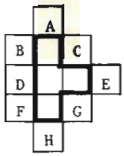
19. At the beginning of a trip, the mileage odometer read 56,200 miles. The driver filled the gas tank with 6 gallons of gasoline. During the trip, the driver filled his tank again with 12 gallons of gasoline when the odometer read 56,560. At the end of the trip, the driver filled the tank again with 20 gallons of gasoline. The odometer read 57,060. To the nearest tenth, what was the car's average miles-per-gallon for the entire trip?

A) 22.5 B) 22.6 C) 24.0 D) 26.9 E) 27.5

20. The value of the expression $\frac{(304)^5}{(29.7)(399)^4}$ is closest to

A) .003 B) .03 C) .3 D) 3 E) 30

21. Suppose one of the eight lettered identical squares is included with the four squares in the T-shaped figure outlined. How many of the resulting figures can be folded into a topless cubical box?



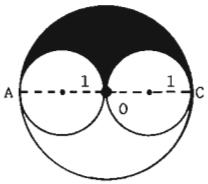

A) 2 B) 3 C) 4 D) 5 E) 6

22. Alan, Beth, Carlos, and Diana were discussing their possible grades in mathematics class this grading period. Alan said, "If I get an A, then Beth will get an A." Beth said, "If I get an A, then Carlos will get an A." Carlos said, "If I get an A, then Diana will get an A." All of these statements were true, but only two of the students received an A. Which two received A's?

A) Alan,Beth B) Beth,Carlos C) Carlos,Diana

D) Alan,Diana E) Beth,Diana

23. The large circle has diameter AC. The two small circles have their centers on AC and just touch at O, the center of the large circle. If each small circle has radius 1, what is the value of the ratio of the area of the shaded region to the area of one of the small circles?




A) between $\frac{1}{2}$ and 1 B) 1 C) between 1 and $\frac{3}{2}$

D) between $\frac{3}{2}$ and 2 E) cannot be determined from the information given

24. The 600 students at King Middle School are divided into three groups of equal size for lunch. Each group has lunch at a different time. A computer randomly assigns each student to one of the three lunch groups. The probability that three friends, Al, Bob, and Carol, will be assigned to the same lunch group is approximately

A) $\frac{1}{27}$ B) $\frac{1}{9}$ C) $\frac{1}{8}$ D) $\frac{1}{6}$ E) $\frac{1}{3}$

25. Which of the following sets of whole numbers has the largest average?

A) multiples of 2 between 1 and 101 B) multiples of 3 between 1 and 101

C) multiples of 4 between 1 and 101 D) multiples of 5 between 1 and 101

E) multiples of 6 between 1 and 101

## SOLUTIONS

A 1986 Solutions Pamphlet will be mailed to your school Examination Manager along with your results.

## WRITE TO US!

Address questions and comments about the problems and solutions for this AJHSME to:

Professor Thomas Butts, AJHSME Chairman
Science Education Department
The University of Texas at Dallas
P.O. Box 830688 FN 32
Richardson, TX 75083-0688

Comments about administrative arrangements and orders for any of the publications listed below should be addressed to:

Professor Walter E. Mientka, CAMC Executive Director
Department of Mathematics & Statistics
University of Nebraska
Lincoln, NE 68588-0322

## 1987 AHSME

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AHSME, 1972-86, Spanish editions, 1978-86, 50¢ per copy per year.
AJHSME National Summary of Results and Awards, 1985, $3.00 per copy per year.
AHSME National Summary of Results and Awards, 1976-86, $3.00 per copy per year.

BOOKS

Contest Problem Book I ($7.50), AHSME exams and solutions, 1950-60.
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