AJHSME SOLUTIONS PAMPHLET

FOR STUDENTS AND TEACHERS

1st ANNUAL
AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION
1985

Sponsors:

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This Solutions Pamphlet gives at least one solution for each problem on this year's Examination and shows that all the problems can be solved using material normally associated with the seventh and eighth grade mathematics curriculum. The solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

It is hoped that teachers will find the time to share these solutions with their students.

AMERICAN MATHEMATICS COMPETITIONS

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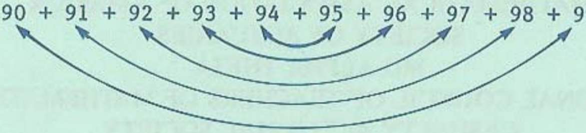
1. (A) $\dfrac{3\times 5}{9\times 11}\times\dfrac{7\times 9\times 11}{3\times 5\times 7}=\dfrac{3\times 5\times 7\times 9\times 11}{3\times 5\times 7\times 9\times 11}=1.$

2. (B) By estimating, we see that the desired sum is between $10\times 90=900$ and $10\times 100=1000$, so it must be 945.

OR

Pair the numbers as shown. The sum of each pair is 189, so the desired sum is $5\times 189=945$.

$$90+91+92+93+94+95+96+97+98+99$$



3. (D) $\dfrac{10^7}{5\times 10^4}=\dfrac{10\times 10^6}{5\times 10^4}=2\times 10^2=200.$

OR

$$\frac{10^7}{5\times 10^4}=\frac{10^3}{5}=\frac{1000}{5}=200$$

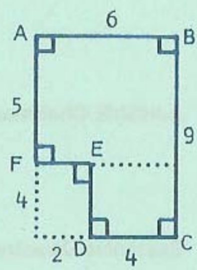
4. (C) The area is greater than $6\times 5=30$ and less than $6\times 9=54$ so (C) must be correct.

OR

Extending FE partitions the polygon into a rectangle and a square whose areas are 30 and 16 respectively.

OR

Extending AF and DC to form the large rectangle shows that area is $(6\times 9)-(4\times 2)=54-8=46$.



5. (C) By reading the graph, there are 5 A's, 4 B's, 3 C's, 3 D's, and 5 F's. Thus the fraction of satisfactory grades is $\frac{5+4+3+3}{20} = \frac{15}{20} = \frac{3}{4}$

OR

By reading the graph, $\frac{5}{20} = \frac{1}{4}$ of the grades are not satisfactory so $1 - \frac{1}{4} = \frac{3}{4}$ of the grades are satisfactory.

6. (D) The 7.5 cm stack is "half again" as tall as the 5 cm stack, so it will contain $500 + \frac{1}{2}(500) = 500 + 250 = 750$ sheets.

OR

If n is the number of sheets of paper in the 7.5 cm stack, then $\frac{5}{500} = \frac{7.5}{n}$. Thus n = 750 sheets.

7. (C) The number of black squares is one less than the number of the row, so the 37th row contains 36 black squares.

8. (A) If a = −2, the set is {6, −8, −12, 4, 1} so 6 = 3a is the largest. Notice that 4a and $\frac{24}{a}$ could be eliminated immediately since they are negative if a is negative.

9. (A) The desired product equals $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \ldots \times \frac{8}{9} \times \frac{9}{10} = \frac{1}{10}$
Notice that since $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$, the product is less than $\frac{1}{3}$ so (C), (D) and (E) are easily eliminated.

10. (C) $$\frac{\frac{1}{3} + \frac{1}{5}}{2} = \frac{\frac{8}{15}}{2} = \frac{4}{15}$$

11. (E) If face X is placed on the bottom of the cube, then faces U, V, W and Z are the sides and face Y is the top.

12. (B) The perimeter of the triangle and the square is $6.2 + 8.3 + 9.5 = 24$ cm. Thus the length of the side of the square is 6 cm and the area is 36 cm^2.

13. (B) To keep the units of miles and hours, first note 45 minutes $= \frac{45}{60} = \frac{3}{4}$ hour and 30 minutes $= \frac{1}{2}$ hour. Since distance = rate × time, your total distance is

$$4 \times \frac{3}{4} + 10 \times \frac{1}{2} = 3 + 5 = 8 \text{ miles.}$$

The distance is less than $4 + 10 = 14$ miles, so (D) and (E) can be easily eliminated.

14. (B) The difference is .5% of \$20 = .005 × \$20 = \$.10.

15. (C) In addition to the 100 numbers from 200-299, there are 20 numbers ending in 2 (e.g., 112, 342) and 20 numbers with a ten's digit of 2 (e.g., 127, 325). But the numbers 122 and 322 are counted twice in this process, so there are a total of $100 + 20 + 20 - 2 = 138$.

16. (D) Since the ratio is 2:3, $\frac{2}{5}$ of the students are boys and $\frac{3}{5}$ of them are girls. Thus there are $\frac{1}{5}$ more girls than boys and $\frac{1}{5} \times 30 = 6$.

17. (D) To get an average of 85 on 7 tests, you needed a total of $7 \times 85 = 595$ points. After 6 tests, you had a total of $6 \times 84 = 504$ points. Thus you needed $595 - 504 = 91$ points on the seventh test.

OR

If n was your score on the seventh test, then $\frac{6(84) + n}{7} = 85$ so $n = 91$.

OR

To raise your average by one point, you needed seven additional points on the seventh test, so your score was $84 + 7 = 91$.

18. (E) Only (E) satisfies the hypothesis that ten copies of the pamphlet cost more that \$11.00.

OR

If P is the price of the pamphlet, then $9P < 10$ and $10P > 11$ or $1.10 < P < 1.1111\ldots$. Thus $P = \$1.11$.

19. (B) If $2(\ell + w)$ is the original perimeter, then the new perimeter is $2(1.1\ell + 1.1w) = 2.2(\ell + w)$ which is 10% more than $2(\ell + w)$.

20. (C) January has 31 days. Had January 1 fallen on a Monday or Tuesday, then there would have been five Tuesdays - 2, 9, 16, 23, 30 or 1, 8, 15, 22, 29. Likewise, had January 1 fallen on a Friday or Saturday, there would have been five Saturdays. Thus (C) is correct.

21. (E) If the initial salary is thought of as \$100 then the first 10% increase gives \$110. The second 10% increase gives \$110 + \$11 = \$121. The third increase gives \$121 + \$12.10 = \$133.10 and the fourth increase gives \$133.10 + \$13.31 = \$146.41 for an increase of 46.41%

OR

If S is the initial salary then the salary after four 10% increases is $(1.1)(1.1)(1.1)(1.1)S = 1.4641S$ for an increase of 46.41%.

22. (B) There are 10 digits. Excluding 0 and 1 leaves 8 digits. Thus $\frac{1}{8}$ of all telephone numbers begin with 9. Of these, $\frac{1}{10}$ end with 0 giving $\frac{1}{8} \times \frac{1}{10} = \frac{1}{80}$ which begin with 9 and end with 0.

23. (E) There are $1200 \times 5 = 6000$ times a day a student attends a class. Thus there are $\frac{6000}{30} = 200$ times a day a teacher teaches a class, so there must be $\frac{200}{4} = 50$ teachers.

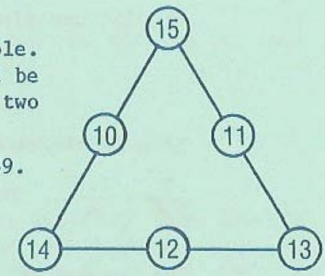
OR

If each student took 4 classes and each teacher taught 4 classes, then $\frac{1200}{30} = 40$ teachers would be required. But each student takes 5 classes, so $\frac{5}{4} \times 40 = 50$ teachers are needed.

24. (D) If the sum S is as large as possible. then the three largest numbers should be at the vertices so they each occur in two sums S. Thus

$$S = \frac{2(13 + 14 + 15) + 10 + 11 + 12}{3} = 39.$$

One such triangle is indicated.



25. (A) If Jane is wrong, then there is a card with a vowel on one side and an odd number on the other side. Such a card cannot have a consonant or an even number on either side. Thus the only card which could prove Jane wrong is the one with a "3" on one side. The other side would have to be a vowel.

OR

The easiest way for Mary to show Jane was wrong is for her to turn over a card showing a vowel, but there is no such card. But if Jane were correct, then so is the contrapositive: "If an odd number is on one side of a card, then a consonant is on the other side." Thus, Mary showed Jane was wrong by turning over the card marked with a 3.